



O.R. Applications

Integrating purchasing and routing in a propane gas supply chain

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Abstract

This paper addresses the integration of purchasing and routing for a propane gas supply chain. The focus is on the wholesale side of the supply network. Rigorous solution methods using both set partitioning and tabu search are developed for obtaining optimal and near-optimal solutions for the purchasing/routing problem. The proposed methods are applied to a real-world propane distribution problem. The results of the study indicate that the potential annual cost savings through the integration of purchasing and routing decisions can range up to millions of dollars for large distributors. The methods can also be used strategically as an aid in reconfiguring the supply network involving depot locations, tanker fleet sizing, and allocation of capacity at supply terminals. The methodology could also be used to provide decision support for distributing other energy products such as heating oil.

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1. Introduction

Propane gas is a major energy source for many commercial and residential users in the United States. In addition to its use in home heating, propane is sold for commercial, industrial, agricultural, and recreational uses. The typical supply chain for propane distribution is a four-level system consisting of propane producers, regional supply terminals, distributor-owned storage plants, and customer storage tanks. Propane is shipped from producers to regional supply terminals via rail, pipeline, or truck. Propane distributors are responsible for purchasing and transporting propane from regional supply terminals to their own storage plants. These plants then serve as the supply sources for the retail distribution to both residential and industrial retail customers.

For propane distributors, the improved coordination and optimization of wholesale operations can yield significant benefits. The largest distributor has an annual sales volume of more than one billion gallons, and

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the second largest distributor serves more than 800,000 customers. Annual distribution costs typically exceed \$50 million dollars for the larger propane distributors.

Previous work in the literature has focused on the inventory/routing problem which involves plant replenishment strategies and the transportation of gas from storage plants to retail customer tanks. The retail side of industrial gas distribution has been reported by Bell et al. (1983). They developed a large-scale, mixed-integer program for solving the inventory/routing problem for Air Products and Chemicals, Inc. Golden et al. (1984a,b) developed an efficient Clarke–Wright-based method to address the retail inventory/routing problem for the distribution of liquid propane to both residential and industrial plants. Dror and Trudeau (1988) examined the relationship between stockouts and route failures for the inventory/routing problem of a propane firm in Pennsylvania. Brown and Graves (1981) modeled a real-time, single-terminal dispatching system of petroleum tank trucks for Chevron Marketing. They solved a sequence of embedded network flow problems in order to minimize transportation costs of refined petroleum products while maintaining equitable personnel and equipment workloads, plant service, and equipment compatibility restrictions. Brown et al. (1987) extended the single-terminal dispatching system to a multiple-terminal dispatching system. The computer assisted dispatch (CAD) also incorporated other functions, such as the consideration of product laid-down costs, loading tanks to maximum weight, split loads, and provision for terminals without trucks. CAD used a collection of integer programming methods to control the real-time distribution of Mobil Oil's light petroleum products. Smith and Moses (1996) used capacitated gravity models to transportation services for petroleum products.

The wholesale side of the petroleum supply chain has been examined by Bausch et al. (1995). They developed an optimization model for the dispatching of Mobil Oil Corporation's heavy petroleum products. Ronen (1995) completed a comprehensive review on a variety of environments for the dispatching of petroleum products and reported on the types of operations research tools that were used by oil companies. Because of the complexity of the dispatching environment, heuristics were the dominant tools used to derive acceptable dispatches and the only viable optimization tools are set partitioning based models. Ronen described the various forms of set partitioning (SP) models which include the basic set partitioning, elastic set partitioning (ESP), and set packing formulations. Ross et al. (1998) have looked at the benefits of reconfiguring the supply network of CountryMark Cooperative, Inc., the largest midwest distributor of agricultural fuels. Russell and Challinor (1988) used a greedy heuristic with an embedded network flow algorithm to dispatch petroleum tank trucks for the pickup and delivery of crude oil to collection terminals.

In this paper, we address the integration of propane gas purchasing and routing. Bramel and Simchi-Levi (1997) have examined integrated logistics models and Anily and Bramel (1999) have worked on vehicle routing methods and the supply chain. However, neither has developed models for integrating purchasing and routing. Our propane distribution problem is related to the complex multiple-product dispatching problem of Brown et al. (1987). We focus on the purchasing/routing aspect of the wholesale portion of the propane supply chain. The objective in the purchasing/routing problem is to minimize the combined costs of purchasing and transportation. The price of propane gas at various supply terminals varies much like gasoline or jet fuel prices. With large tankers holding several thousand gallons, the purchasing and routing strategy can have a major impact on reseller profitability. A basic decision involves how far to travel in order to purchase thousands of gallons of propane at a lower price. Tradeoffs exist between purchase price, travel cost, plants visited, and Department of Transportation (DOT) regulations on driver miles and time.

In the sections that follow, we describe the environment of the purchasing/routing problem. We formulate a model of the problem and discuss relevant properties and characteristics. We then develop set partitioning and tabu search solution approaches for achieving effective solutions to the proposed purchasing routing problem. Finally, we report results pertaining to cost reductions and computational requirements.

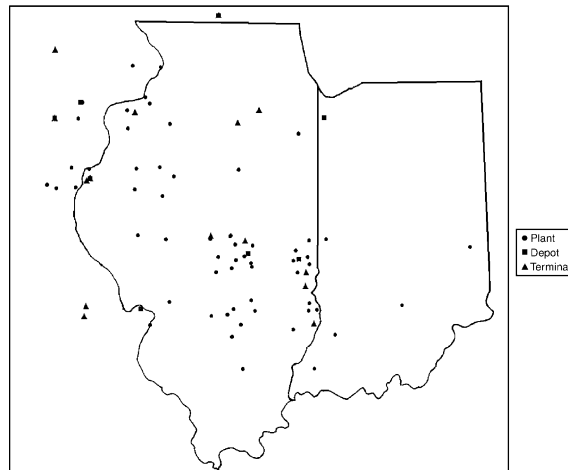


Fig. 1. Illustration of Illinois dispatch area.

2. The purchasing/routing problem

The purchasing/routing problem examined in this paper concerns GASCO,¹ one of the larger propane distributors in the US. The distributor delivers approximately 350 million gallons per year to 365,000 retail demand sites. The retail system is supported by a wholesale logistics system consisting of six major dispatch areas. There are 255 storage plants, 20 truck depots, and 73 supply terminals in these dispatch areas. The Illinois dispatch area is one of the two examined in the study and consists of 88 plants, 29 supply terminals, and seven truck depots housing a total of 16 tanker trucks. Fig. 1 shows the Illinois dispatch area and the spatial characteristics of the problem.

A typical dispatch area is responsible for maintaining adequate inventory levels at its storage plants. These plants typically stock 30,000 gallons and are used to supply customer retail tanks. A plant is eligible for a truckload delivery if it can hold a full truckload of 9200 gallons. Plants are scheduled for delivery based on the current inventory level, capacity, and forecasted demand usage. During the winter months, some plants might require two or three truckload deliveries in one day. For this reason, delivery time windows might need to be imposed to ensure that tanker deliveries are sufficiently separated in time, i.e., morning delivery, afternoon delivery, etc. Delivery time windows introduce a restriction on when deliveries are allowed. Arrivals before the delivery window result in tanker waiting. These time windows are somewhat “soft”, but were treated as “hard” time windows. Thus, deliveries after the delivery window are considered as infeasible.

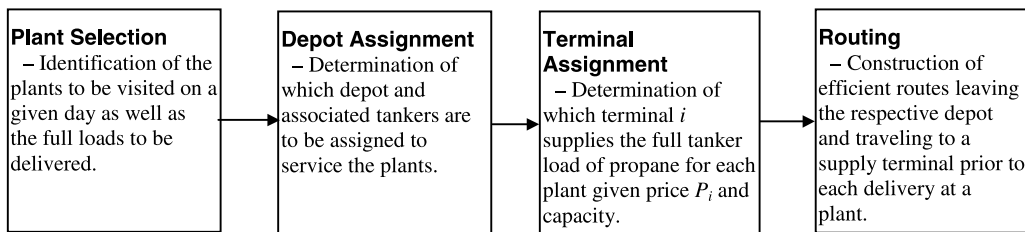
The planning horizon for dispatch decisions is 24–36 hours. Many of the delivery tankers are used around the clock by utilizing two 12-hour shifts and different drivers. DOT regulations impose constraints on maximum miles driven (500), hours spent driving (10), and total work time (15 hours) per day. Additionally there are maximum total driver work hour limits per week. The supply terminals are propane sources where the tankers fill up prior to delivery. Some supply terminals have multiple vendors with various prices for propane at the same location. In some cases, the distributor has a contract with a particular vendor at a supply terminal; in other cases, the distributor is free to purchase from any vendor.

¹ GASCO is a fictitious name for a real company.

Distributors sometimes enter into supply contracts in advance to hedge against future gas price movements during the peak winter months.

The primary bases upon which to select a supply terminal are price and proximity. However, during the peak season, a secondary factor is the queue size and waiting time at a terminal. A tanker requires approximately an hour to load or unload. In extremely cold weather, demand is high, waiting lines are often long, and waiting times at some terminals can be as long as six hours. Although the proposed model does not explicitly address queuing behavior, it is possible to limit the total number of tankers assigned to a terminal.

Demand on a given day can exceed the capacity of the distributor. In these cases, it is possible to supplement deliveries with common carrier tankers. Common carriers, however, cost significantly more per mile than the distributor-owned tankers at the daily operational level. The primary decisions involved in the purchasing/routing problem for propane distribution can be described by the following flow diagram:



A decision support system for the purchasing/routing problem can also be applied to longer-term strategic problems. Decisions regarding the location of depots and the optimal number of tankers to assign to each depot can be addressed through the purchasing/routing methodology proposed. We turn now to the formulation of a mathematical model to clearly describe the purchasing/routing problem.

3. The formulation of purchasing/routing propane gas logistics problem

Here we formally describe the purchasing/routing propane gas logistics problem which is a general multi-depot vehicle routing problem. The problem can be defined as a directed graph $G = \langle \Pi, E \rangle$. Π is the set of the nodes in the graph and $\Pi = P \cup T \cup D$, where P is the set of plants, T is the set of terminals (suppliers), D is the set of depots. E is the set of the edges in the graph and $E = E_1 \cup E_2 \cup E_3 \cup E_4$, where $E_1 = \{ \langle x, y \rangle | x \in D \text{ and } y \in T \}$, $E_2 = \{ \langle x, y \rangle | x \in T \text{ and } y \in P \}$, $E_3 = \{ \langle x, y \rangle | x \in P \text{ and } y \in T \}$, and $E_4 = \{ \langle x, y \rangle | x \in P \text{ and } y \in D \}$. Each tanker starts from a depot and travels alternatively between $n - 1$ terminals and $n - 1$ plants (picking up and delivering full tanker loads) where n is an arbitrary number. At the end of the route, it returns to the same depot. Therefore, for any route, the sequence of the nodes is $\langle \pi_1, \pi_2, \pi_3, \dots, \pi_{2n} \rangle$, where $\pi_1 = \pi_{2n} \in D$, $\pi_{2i} \in T$ ($1 \leq i \leq n - 1$), $\pi_{2i+1} \in P$ ($1 \leq i \leq n - 1$). At each plant, there is a time window in which service must start. Route duration is restricted according to federal law. The total cost of the problem includes the purchase cost of gas and the transportation cost. The objective of the problem is to build a set of routes in such a way that all of the following constraints are satisfied:

- For each route, the tanker starts and ends at the same depot.
- For each route, the time duration must be less than or equal to the preset limit.
- Each plant node is served only once (plants requiring multiple deliveries are represented as multiple nodes). The start time during which the plant is served must be within its time window.
- Prior to delivery, the tanker must travel to one of the terminals to purchase gas.
- The total cost, including travel and gas purchase cost, is to be minimized.

To ensure that the model has a feasible solution, we assume that there is enough capacity to service all plants. In other words, there is sufficient capacity of gas at terminals and sufficient number of tankers available to service all plants.

Parameters

Π	set of plants, depots, and terminals
D	set of depots
P	set of plants
T	set of supply terminals
p_i	price per gallon of propane gas at supply terminal i , $i \in T$
q_i	capacity at terminal i , $i \in T$
V_d	set of tankers at depot d , $d \in D$
V	set of tankers at all depots, i.e., $V = \bigcup_{d \in D} V_d$
C_k	capacity of tanker k , $k \in V$
β	travel cost per mile
d_{ij}	distance between i and j , $i, j \in \Pi$
TD	total time duration allowed per route per day.
TM	total mileage allowed per route per day
s_i	service time at plant i or load time at terminal i , $i \in P \cup T$.
t_{ij}	travel time between i and j , $i, j \in \Pi$
Z_k	time constraint for tanker k , $k \in V$
W_{i1}	earliest time of time window at plant i , $i \in P$
W_{i2}	latest time of time window of plant i , $i \in P$

Variables

X_{ij}^k	$\begin{cases} 1 & \text{if vehicle } k \text{ travels from } i \text{ to } j \text{ where } k \in V \text{ and } i, j \in \Pi \\ 0 & \text{otherwise} \end{cases}$
e_i^k	arrival time of tanker k at i where $k \in V$ and $i \in P$

Model:

$$\min \sum_{k \in V} \sum_{j \in P} \sum_{i \in T} X_{ij}^k (p_i C_k) + \sum_{k \in V} \sum_{i \in \Pi} \sum_{j \in \Pi} \beta d_{ij} X_{ij}^k \quad (1)$$

s.t.

$$\sum_{k \in V} \sum_{j \in D \cup T} X_{ij}^k = 1, \quad i \in P, \quad (2)$$

$$\sum_{k \in V} \sum_{i \in T} X_{ij}^k = 1, \quad j \in P, \quad (3)$$

$$\sum_{j \in P} X_{ij}^k - \sum_{j \in P \cup D} X_{ji}^k = 0, \quad i \in T, \quad k \in V, \quad (4)$$

$$\sum_{i \in D} \sum_{j \in T} X_{ij}^k - \sum_{j \in P} \sum_{i \in D} X_{ji}^k = 0, \quad k \in V_d, \quad d \in D, \quad (5)$$

$$\sum_{j \in T \cup D} X_{ij}^k - \sum_{j \in T} X_{ji}^k = 0, \quad i \in P, \quad k \in V, \quad (6)$$

$$\sum_{k \in V_d} \sum_{i \in P} X_{id}^k = |V_d|, \quad d \in D, \quad (7)$$

$$\sum_{k \in V} \sum_{j \in P} C_k X_{ij}^k \leq q_i, \quad i \in T, \tag{8}$$

$$\sum_{i \in \Pi} \sum_{j \in \Pi} d_{ij} X_{ij}^k \leq TM, \quad k \in V, \tag{9}$$

$$\sum_{i \in \Pi} \sum_{j \in \Pi} (t_{ij} + s_i) X_{ij}^k \leq TD, \quad k \in V, \tag{10}$$

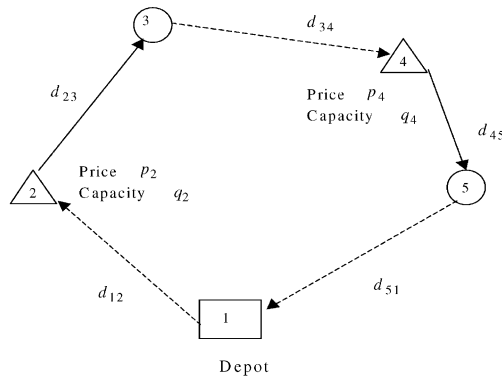
$$\sum_{i \in \Pi} (e_i^k + t_{ij} + s_i) X_{ij}^k \leq e_j^k, \quad k \in V_d, \quad d \in D, \quad j \in P \cup T, \tag{11}$$

$$W_{i1} \leq e_i^k \leq W_{i2}, \quad k \in V_d, \quad d \in D, \quad i \in P, \tag{12}$$

$$X_{ij}^k \in \{0, 1\}, \quad i \in \Pi, \quad j \in \Pi, \quad k \in V. \tag{13}$$

The objective function (1) minimizes the total travel and gas purchasing cost. The β cost factor includes the variable operating expenses such as driver salaries and fuel cost expenses in dollars per mile. In the GASCO application the management treated the tankers used as a sunk cost. In other applications the explicit cost of using tankers could be considered. Fig. 2 illustrates the cost calculation for a hypothetical route for a tanker with capacity $C_k = 9200$ gallons and $\beta = \$1.50$ per mile driven; β does not include a cost per hour. Constraints (2) and (3) assign a plant to a tanker route. Constraints (4)–(6) are the flow conservation constraints for terminals, depots, and plants, respectively. Constraints (7) ensure that the number of tankers utilized cannot exceed the total number of tankers stationed at their respective depots. Constraints (8) ensure that demand at any supply terminal would not exceed its capacity. Constraint (9) ensures that route length will not exceed TM miles (500 miles in our application) per day. Constraints (10) ensure that route duration will not exceed TD (typically 720 minutes) per day. Constraints (11) and (12) are time window constraints. Finally, constraints (13) are the binary constraints.

The integer formulation states the objective and constraints associated with the purchasing/routing problem. However, the model is computationally intractable since most applications would involve several thousand binary decision variables and constraints (11) are nonlinear. Therefore, for real-world applications, we focus our attention on the development of an optimization-based set partitioning approach and an effective tabu search metaheuristic.



The total routing cost is: $9200p_2 + 9200p_4 + 1.50(d_{12} + d_{23} + d_{34} + d_{45} + d_{51})$

Fig. 2. Route cost calculation.

4. Solution approaches

4.1. Set partitioning approach

We have developed two solution approaches to solve the purchasing/routing problem. We consider both a SP model and a tabu search metaheuristic. SP has been used in a variety of vehicle routing and distribution applications. Bramel and Simchi-Levi (1997) examined the effectiveness of SP in solving vehicle routing problems with time windows. They observed that for any distribution of service times, time windows, customer loads, and locations, the relative gap between fractional and integer solutions of the set covering problem becomes arbitrarily small as the number of customers increases. Ronen (2000) used ESP to generate solutions within 1% of optimality for scheduling charter aircraft.

SP is an optimization-based approach that is capable of not only handling nonlinear and fixed route costs, but is also capable of generating optimal solutions provided the problem size is not too large. In order to guarantee an optimal solution, all feasible routes must be considered implicitly or explicitly in the integer programming formulation. To generate an approximate or heuristic solution, a subset of all possible routes can be generated in order to reduce problem size and computation time. The optimization approach to using a set partitioning formulation is as follows:

1. Generate all possible routes.
2. Preprocess the routes to eliminate all infeasible routes.
3. Solve the resulting 0–1 integer SP model.

In step 1, all possible routes were generated by determining all combinations of routes containing one, two, three, or four plants; five plant deliveries on a single route was never feasible. All route permutations and all possible assignments of terminals to plants were generated. In the largest test problem, more than one trillion routes were generated.

In step 2, feasibility is determined by applying DOT criteria pertaining to maximum daily travel distance (500 miles) and maximum route duration (12 or 15 hours). Preprocessing greatly reduced the number of feasible routes. The largest number of feasible routes for any test problem was 424,499. Step 3 was achieved by solving the associated 0–1 SP integer model.

A heuristic approach using SP is developed by considering only the minimal-distance combination of plants on a route. For this plant sequencing on a route, all possible depots and supply terminal assignments are generated. This state space reduction results in a significant problem size reduction with very little effect on solution quality. For the ten test problems, the optimization approach required a minimum of 30,998 to a maximum of 424,499 0–1 variables (feasible routes). The heuristic approach reduced the number of 0–1 variables to a range of 7066–65,091. Additionally, the preprocessing time required to generate the MPS file for input to the IBM OSL solver was reduced from a range of 1.1 minutes to 11.5 hours on a 1.32 GHz computer for the optimization approach to a range of 0.4 minutes to 1.4 hours for the heuristic approach.

The generalized SP model is formulated as follows:

$$\begin{aligned} \min \quad & z_r X_r \\ \text{s.t.} \quad & \\ & \sum_{r \in R} a_{ir} X_r = 1, \quad \forall i \in P, \end{aligned} \tag{14}$$

$$\sum_{r \in R_d} X_r \leq |R_d|, \quad \forall d \in D, \tag{15}$$

$$\sum_{r \in R} u_{jr} X_r \leq q_j, \quad \forall j \in T, \tag{16}$$

$$X_r = \begin{cases} 1 & \text{if route } r \text{ is used,} \\ 0 & \text{otherwise.} \end{cases}$$

Parameters used are as follows:

z_r	cost of route r ,
a_{ir}	$\begin{cases} 1 & \text{if route } r \text{ covers plant } i, \\ 0 & \text{otherwise,} \end{cases}$
R	set of all routes,
D	set of all depots,
$ R_d $	total number of routes at depot d ,
u_{jr}	gallons obtained from terminal j used in route r ,
q_j	capacity of terminal j .

The first constraint set (14) are “covering” type constraints that ensure that each plant is served by exactly one vehicle. The second set of constraints (15) ensure that the total number of vehicles used at a depot does not exceed the number of vehicles available at that depot. The last set of constraints (16) enforces the available gas capacity at each supply terminal.

The resulting number of constraints is small relative to the number of feasible routes in the SP formulation. No more than 60 constraints were generated in the pilot study test problems. The small number of constraints fostered the computational tractability of the set partitioning approach. The resulting 0–1 integer models were solved using Version 3 of IBM’s OSL MIP solver. Computing time for the OSL MIP solver ranged from 2.4 hours to termination after 60 hours.

4.2. Tabu search approach

The proposed metaheuristic is a two-phase approach consisting of a parallel route construction procedure and a tabu search route improvement algorithm. Tabu search has been successfully applied to various vehicle routing applications. Gendreau et al. (1994) developed a tabu search heuristic for the vehicle routing problem with capacity and route length restrictions. Rego and Roucairol (1995) applied tabu search to generate strong results for a dynamic, multi-terminal, tank dispatching problem. Taillard et al. (1996) developed an efficient tabu search heuristic to solve vehicle routing problems with 40–361 plants. Renaud et al. (1996) proposed a tabu search algorithm for the multi-depot vehicle routing problem with capacity and route length restrictions. Brando and Mercer (1997) presented a tabu search based heuristic for solving real-world distribution problems pertaining to multi-trip vehicle routing and scheduling problems. Chiang and Russell (1997) developed a reactive tabu search heuristic for the vehicle routing problem with time windows.

The route construction procedure is “parallel” in that all routes are constructed simultaneously rather than one at a time. In the construction phase, supply terminals are initially assigned to plants using a transportation network flow model. An important parameter is the price–distance parameter which regulates the distance a tanker will travel to purchase less expensive gas. The parameter, α , is utilized in the network flow objective function coefficients c_{ij}^k (where c_{ij}^k is the “cost” of assigning a full load from terminal i to plant j via tanker k) as follows:

$$c_{ij}^k = \alpha d_{ij} + C_k p_i$$

without the price–distance parameter, the plants will tend to be assigned to terminals too far away resulting in insufficient tankers to service the plants on a given day. The gas cost of a full tanker dominates the travel cost. The parameter assumes a range of five arbitrary values ($\alpha = 2, 4, 6, 10, 100$) to create five passes in the solution process. A value of $\alpha = 2$ will emphasize low gas cost sometimes at the expense of travel distance,

time, and the number of tankers required. A value of $\alpha = 100$ emphasizes efficiency in terms of total travel miles and number of tankers required.

4.3. Initial solution construction

The parallel construction heuristic is composed of two parts: (1) Insertion of unrouted plants to construct new routes. (2) Improvement both during construction and after generation of the initial solution. The steps of the heuristic are as follows:

4.3.1. Construction of routes by insertion of unrouted plants

1. Initially assign plants to terminals using a transportation network flow model with cost element $c_{ij}^k = \alpha d_{ij} + C_k p_i$. Each plant has a demand of exactly one full tanker load and each terminal has a supply equal to its current gas inventory in number of full tanker loads.
2. Sequentially assign plants to emerging routes.
3. For each unrouted plant u , its best insertion place in each of $|R|$ routes is based on a time measure. For each route r , $r = 1, 2, \dots, |R|$, to insert u between i_r and j_r , calculate $c_{1r} = b_{uj} - b_j$, where b_j is the current time that service can begin at point j ; b_{uj} is the new time that service can begin at j given that u is now inserted in the route.

Route r^* is chosen for which

$$c_{1r^*}(i_{r^*}(u), u, j_{r^*}(u)) = \min[c_{1r}(i_r(u), u, j_r(u))], \quad r = 1, 2, \dots, |R|.$$

4.3.2. Improvement during construction

1. Improve single routes using a modified k -opt traveling salesman algorithm based on the Lin and Kernighan (1973) TSP heuristic. The route improvement algorithm is invoked after every three plants are added.
2. Call tabu search for global route improvement after every 10 plants have been routed. Use the same pseudo objective function, $c_{ij}^k = \alpha d_{ij} + C_k p_i$ to evaluate neighborhood moves.
3. Invoke the final application of the tabu search route improvement procedure after attempting to route all plants.

4.4. Neighborhood definition

Tabu search is an iterative heuristic procedure exploring the solution space from the current solution to its neighborhood. In our application, the neighborhood of one solution is defined as the set of the solutions which can be reached by three primary plant movements: insert movement, delete movement, and exchange movement. The primary move is enhanced by a secondary terminal adjustment. This primary plant movement has been called the λ -interchange mechanism and is attributed to Osman (1993) and Osman and Salhi (1996). We implement $\lambda = 1$ in which one plant at a time is moved or exchanged between routes. After the primary plant movement is executed, we perform a secondary exhaustive search for the best terminal to serve the moved plant and adjacent plants on the affected routes.

4.5. Search moves

The format of the insert movement is $\text{insert}(r_1, r_2, i, j)$. It moves a plant from route r_1 position i to route r_2 position j . Then reassign the supplier terminals based on the combined costs of transportation and purchasing (see Property 1 for further explanation). We label insert movement as a $(0, 1)$ movement.

The format of the delete movement is $\text{delete}(r_1, r_2, i, j)$. It moves the plant from the r_2 position j to r_1 position i . The supply terminals in both r_1 and r_2 also need to be reassigned based on transportation and purchasing costs. The delete movement is also called a $(1, 0)$ movement. It is easy to check that the $(0, 1)$ and $(1, 0)$ are a pair of reverse movements.

The third movement is an exchange movement. The format of the exchange movement is $\text{exchange}(r_1, r_2, i, j)$. It exchanges the plants in r_1 position i and r_2 position j . The exchange movement is labeled as a $(1, 1)$ movement. The supply terminals before and after the exchanged plants also need to be reassigned.

Not every movement is feasible. The feasibility for the movements of plants is dictated by time windows and the distance and time duration of a route. The following lemma incorporates the earliest service start time and the latest service start time of a plant to determine the feasibility of a movement.

The earliest service start time of a plant is the earliest time when the tanker can unload at the plant. The recursive algorithm for the earliest start time A_i and the departure time B_i given the current route structure is

- (1) $A_0 = 0, B_0 = 0;$
- (2) For $i = 1, 2, \dots, |P|,$

$$A_i = \max\{B_{i-1} + t_{i-1,j,i} + s_j, W_{i,1}\},$$

$$B_i = A_i + s_i,$$

where $t_{i-1,j,i}$ is the traveling time from plant $i - 1$ to plant i through supply terminal j .

The latest service start time is the latest time when the tanker could unload at the plant without violating time windows or the maximum route duration. The recursive algorithm for the latest service start time A'_i and the latest departure time B'_i is

- (1) $A'_n = \min\{W_{n,2}, \text{TD} - t_{nd} - s_n\}, \quad B'_n = A'_n + s_n,$
- (2) For all $i \in P,$

$$B'_{i-1} = \min\{A'_i - t_{i-1,j,i} - s_j, W_{i-1,2} + s_{i-1}\},$$

$$A'_{i-1} = B'_{i-1} - s_{i-1},$$

where TD is the maximum allowable time duration of the route; t_{nd} is the traveling time from the last plant n to the depot. Obviously, a route is feasible if and only if for every plant i in the route, the service start time is within the range of $[A_i, A'_i]$. Therefore, we have the following lemma to check the feasibility of a movement between two routes.

Lemma 1. *Suppose plant vertices i and j represent the i th plant on route r_1 and the j th plant on route r_2 respectively. i and j are exchangeable if and only if the following conditions hold:*

There exist supply nodes u_1 and v_1 such that

$$\max\{B_{i-1} + t_{i-1,u_1,j} + s_{u_1}, W_{j,1}\} \leq \min\{A'_{i+1} - t_{j,v_1,i+1} - s_{v_1}, W_{j,2} + s_j\} - s_j. \tag{17}$$

And there exist supply nodes u_2 and v_2 such that

$$\max\{B_{j-1} + t_{j-1,u_2,i} + s_{u_2}, W_{i,1}\} \leq \min\{A'_{j+1} - t_{i,v_2,j+1} - s_{v_2}, W_{i,2} + s_i\} - s_i. \tag{18}$$

Proof. If i and j are exchangeable, then $A_j \leq A'_j$, where

$$A_j = \max\{B_{i-1} + t_{i-1,u_1,j} + s_{u_1}, W_{j,1}\}, \text{ and } u_1 \text{ is the supply terminal between plants } i - 1 \text{ and } j.$$

$$A'_j = \min\{A'_{i+1} - t_{j,v_1,i+1} - s_{v_1}, W_{j,2} + s_j\} - s_j, \text{ and } v_1 \text{ is the supply terminal between plants } j \text{ and } i - 1.$$

Therefore, there exist u_1 and v_1 such that

$$\max\{B_{i-1} + t_{i-1,u_1,j} + s_{u_1}, W_{j,1}\} \leq \min\{A'_{i+1} - t_{j,v_1,i+1} - s_{v_1}, W_{j,2} + s_j\} - s_j.$$

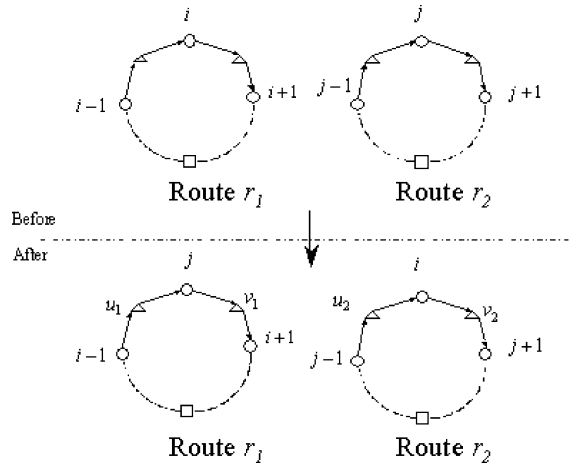


Fig. 3. Feasibility check of plant nodes i and j after exchange.

Similarly, there exist u_2 and v_2 such that

$$\max\{B_{j-1} + t_{j-1,u_2,i} + s_{u_2}, W_{i,1}\} \leq \min\{A'_{j+1} - t_{i,v_2,j+1} - s_{v_2}, W_{i,2} + s_i\} - s_i.$$

For route r_1 , if there exist u_1 and v_1 which satisfy

$$\max\{B_{i-1} + t_{i-1,u_1,j} + s_{u_1}, W_{j,1}\} \leq \min\{A'_{i+1} - t_{j,v_1,i+1} - s_{v_1}, W_{j,2} + s_j\} - s_j,$$

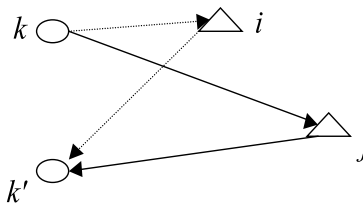
then $A_j \leq A'_j$, i.e., the service start time is in the range of $[A_j, A'_j]$. For the nodes before plant node j , the exchange of nodes between i and j has no impact on them, so they are feasible. For node j , since $A_j \leq A'_j$, j is also feasible. For the nodes following j , by definition, their feasibility requires only that the tanker arrive at j before A'_j . Since the requirement is satisfied, these nodes are also feasible. Therefore, route r_1 is feasible.

In the same way, it can be shown that route r_2 is also feasible. \square

Formulae (17) and (18) ensure that the time duration of both routes and the time window constraints of each plant are satisfied after switching. Fig. 3 illustrates the operation of the exchange.

Among those supply terminals that satisfy formulae (17) and (18), we choose the terminal with the least cost, which includes purchase cost and transportation cost. After switching, we adjust the capacity of supplier terminals. The following property provides a guideline for choosing the right supplier.

Property 1. If $\min_{s,t,s,t \in T} (|p_s - p_t|) > (2\beta/C) \max_{m,n,m,n \in P,n \in T} \{d_{mn}, d_{nm}\}$, where $C = \min_{k \in V} \{C_k\}$, i.e., the minimum capacity of all tankers, then the total purchase price of propane gas dominates the total travel cost. In other words, we will choose a farther supply terminal j over a closer supply terminal i because the price of gas at terminal j is cheaper than that at terminal i .



Proof. Without loss of generality, let us assume that $p_i > p_j$, $i, j \in T$.

Since

$$\begin{aligned}
 p_i - p_j &\geq \min_{\substack{vs, t \\ s, t \in T}} (|p_s - p_t|) > \frac{2\beta}{C} \max_{\substack{\forall m, n \\ m \in P \\ n \in T}} \{d_{mn}, d_{nm}\} \geq \frac{\beta}{C} (d_{kj} + d_{jk'}) > \frac{\beta}{C} (d_{kj} - d_{ki} + d_{jk'} - d_{ik'}) \\
 &= \frac{\beta}{C} (d_{kj} + d_{jk'}) - \frac{\beta}{C} (d_{ki} + d_{ik'}).
 \end{aligned}$$

That is,

$$C(p_i - p_j) > \beta(d_{kj} + d_{jk'}) - \beta(d_{ki} + d_{ik'}).$$

Thus,

$$Cp_i + \beta(d_{ki} + d_{ik'}) > Cp_j + \beta(d_{kj} + d_{jk'}).$$

Therefore, we should choose supply terminal j instead of supply terminal i . \square

Two examples of movements (0, 1) and (1, 1) with and without terminal reassignment are shown in Fig. 4.

Move (0, 1) moves plant 3 from r_2 to r_3 , and the supply terminal is also reassigned. Move (1, 1) switches two plants on two different routes and keeps the accompanying supply terminals unchanged after an evaluation.

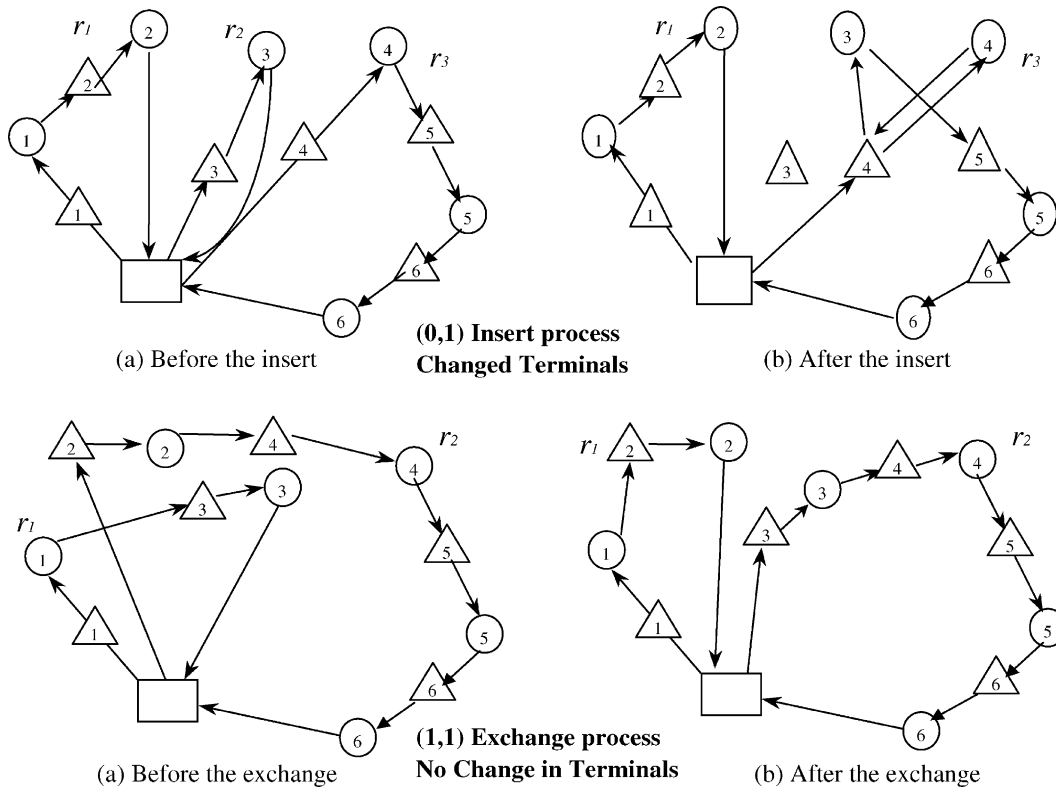


Fig. 4. Illustration of search movements.

4.6. Tabu list structure

A plant is said to have tabu status with respect to a route if it cannot be switched back to that route. Let us define $TABU(u, r), \forall u \in \{1, \dots, |P|\}, \forall r \in \{1, \dots, |R|\}$, where $|P|$ is the cardinality of the set of plants and $|R|$ is the cardinality of the set of routes. $TABU(u, r)$ records the tabu status of plant u in route r , after being moved to another route, i.e., if the $TABU(u, r) = x$, then plant u cannot return to route r until iteration x finishes. Let t be a tabu list size. If plant u switches from route r to r' at the current iteration, then $TABU(u, r)$ is updated as follows:

$TABU(u, r) = \text{iter} + t$ where iter is the current iteration number.

Plant u is tabu w.r.t. route r if $TABU(u, r) > \text{iter}$.

Hence, for operators $(0, 1)$ or $(1, 0)$, if u switches from r_1 to r_2 and if $TABU(u, r_2) > \text{iter}$, then the move is tabu; otherwise, it is not tabu. For operator $(1, 1)$, if plant u in route r_1 is exchanged with plant u' in route r_2 and if $TABU(u, r_2) > \text{iter}$ or $TABU(u', r_1) > \text{iter}$, then the move is tabu; otherwise, it is not. When a plant is tabu, it cannot return to its original route in the next *tabu list size* iterations if it switches from that route to another unless the aspiration criterion is satisfied. The aspiration criterion is as follows: Tabu status can be overridden if the switch of a plant in Tabu can result in a new solution with cost less than the best solution found so far.

4.7. Search intensification

The recency-based memory structure is further enhanced with a longer-term memory to find better solutions. We implement intensification and diversification strategies. In the intensification strategy, we try to keep “elite solutions”, those solutions that significantly improve the total cost. That is, after a certain move the following desirable scenarios occur: a plant’s wait time is eliminated, the number of tankers is reduced by one (a significant savings for a company), or the total cost is reduced by at least $\eta\%$. A wait time occurs when a tanker arrives at a plant earlier than the specified early start time of the time window of the plant. The intensification strategy is designed to reduce the wait time of each plant. An array $INTENSIFY(u)$ is used to record the iteration number that plant u can get out of intensification. When a plant u is in intensification (fixed), it cannot move to another route unless a better solution can be obtained than the best found so far.

4.8. Search diversification

The frequency-based diversification strategy involves a penalty function, which is added to the objective function to penalize those plants that switch too frequently. The penalty function $PF(u_1, u_2)$ is given by

$$PF(u_1, u_2) = \text{move_evaluation}(u_1, u_2) - w^* \text{frequency}(u_1, u_2)$$

where $\text{move_evaluation}(u_1, u_2)$ is the difference of objective function values before and after the move.

$$\text{frequency}(u_1, u_2) = \begin{cases} \text{stime}(u_2) & \text{for operation}(0, 1), \\ \text{stime}(u_1) & \text{for operation}(1, 0), \\ (\text{stime}(u_1) + \text{stime}(u_2))/2 & \text{for operation}(1, 1), \end{cases}$$

$$w = \begin{cases} 0 & \text{if } \text{move_value}(u_1, u_2) < 0, \\ \omega \in [7, 50] & \text{otherwise.} \end{cases}$$

$\text{stime}(u)$ is the number of times that plant u switches position.

It is obvious that when two plants switch too many times, they would be penalized and forbidden to exchange positions in the next specified iterations.

5. Implementation results

In order to test the effectiveness of the proposed solution approaches, pilot studies were conducted in both the Illinois and Michigan dispatch areas. Data were collected for one week in each of these dispatch areas during the month of January, which is typically during the peak season. Relevant data included the daily terminal prices, plant inventory levels, and time and distances between locations. The Illinois dispatch area served a total of 88 plants whose gas could be purchased from a maximum of 29 operating terminals. The seven depots housed tankers in quantities of 1, 3, 2, 5, 1, 2, and 2, respectively. All tankers had a capacity of 9200 gallons. During the week studied, the number of full loads to be delivered on any day varied from a low of 24 to a high of 32. Some plants required more than one load on a given day.

The Michigan dispatch area served a total of 49 plants from three different depot locations and had purchasing options at 12 different terminals/vendors. The three depots housed 14, 3, and 1 tankers, respectively. Most of the tankers were served by two driving crews and could cover two 12 hour shifts or routes per day. The majority of the tankers had a 12,200 gallon capacity, while one depot utilized the smaller 9200 gallon tankers. During the week studied, the number of daily loads ranged from 25 to 40. In the more northern climate, some plants would occasionally require three loads in one 24-hour day. Most of the tankers were scheduled for two 12-hour shifts allowing two routes to be generated per tanker, some tankers were assigned to one 15-hour shift and used only once per 24-hour period. The choice of which tankers to assign to 12-hour shifts as well as which plants to service were determined by GASCO management and treated as inputs in this study. Future research will examine the inventory routing aspect of daily plant selection as well as the optimal number of tankers to utilize at the various depots.

The two dispatch areas generated five test problems each comprised of purchases and deliveries for Monday through Friday of the test week. In order to test the solution procedure on larger problems, three additional test problems were created from the Illinois data set. In these three problems, all 88 plants were assumed to need service once, twice, and three times in a given day. The resulting test problems are labeled D88, D176, and D264, respectively. The size of the test problems (in terms of plants served) ranges up to nine times the typical daily problem.

The tabu search metaheuristic for the purchasing routing problem was coded in FORTRAN, compiled in Microsoft FORTRAN Powerstation, and run on a 1.3 GHz Athlon personal computer. The tabu search was executed with a maximum of 1000 iterations. The solution procedure consists of five passes through the construction/improvement cycle. The price–distance parameter varied in each pass; the values were 2, 4, 6, 10, and 100, respectively.

Table 1 shows the results achieved in the Illinois dispatch area. The table reports the results achieved by tabu search and SP in terms of tankers used and miles driven. The total cost reported is comprised of the propane purchasing cost plus an estimated \$1.50 per mile driven. The gap in the table refers to the percentage increase in total cost relative to the lower bound for all five test problems. The lower bound was determined by solving the LP relaxation of the set partitioning formulation with all possible routes. It should be noted that no fixed cost was associated with the number of tankers used. The GASCO management treated the tankers as a sunk cost. In other applications where the tankers are a relevant cost, incorporating fixed vehicle costs in the objective function would yield more overall cost effective solutions than minimizing only purchasing and travel costs. In fact, the operation of the tankers usually incurs a significant cost. The typical new tanker trailer costs approximately \$90,000 and the associated power unit costs approximately \$60,000. These units have a useful life of 5–7 years. Relevant variable costs include the driver costs of \$40,000 per year (plus 30% for benefits), \$3000 per year for insurance and as much as \$15,000 per year for workman's compensation. Accounting for the tax benefits of depreciation would still yield an annual cost of \$80,000–\$85,000 per year.

Table 1
Results on Illinois dispatch area

Day	Tabu search						SP-heuristic solutions						SP-optimal solutions					Lower bound Total cost (\$)	
	Tanker routes used	Miles driven	Product cost (\$)	Total cost (\$)	Gap (%)	CPU time ^a	Tanker routes used	Miles driven	Product cost (\$)	Total cost (\$)	Gap (%)	CPU Time ^a	Tanker routes used	Miles driven	Product cost (\$)	Total cost (\$)	CPU time ^a		
Monday	11	4412	111,839	118,457	0.93	0.06	12	5358	109,578	117,615.4	0.21	32.7	12	5169	109,774	117,369	144.9	117,369	
Tuesday	11	3927	130,547	136,438	1.10	0.18	12	4955	128,056	135,489.1	0.40	524.6							134,947
Wednesday	11	4453	131,168	137,848	1.02	0.18	12	4919	129,639	137,018.4	0.41	60.58	12	5582	128,693	136,461	2904.8		136,460
Thursday	11	4615	112,859	119,881	1.14	0.12	12	5275	110,886	118,799.2	0.23	118.72							118,524
Friday	13	5007	150,920	158,430	0.95	0.15	13	5516	149,270	157,543.9	0.38	472.5							156,939
Total	57	22,414	637,333	671,054		0.69	61	26,023	627,432	666,466		1209							

^a CPU minutes using a 1.3 GHz Athlon processor.

Tabu search achieved solutions that were within 1.14% of optimality with CPU times averaging less than 9 seconds. On an older data set, tabu search generated solutions that were typically 5% less costly than the GASCO solutions resulting in a weekly savings of approximately \$28,000.

The SP heuristic achieved solutions that were within 0.41% of the lower bound with CPU times ranging from 32.7 to 472.5 minutes. When all route permutations were considered, SP was able to obtain the optimal solution for two Illinois test problems. The CPU time required to determine the two optimal solutions were 144.9 and 2904.8 minutes, respectively. The other three Illinois test problems were terminated after 60 hours of CPU time and had not achieved an optimal solution. The computational requirements of SP would be suitable for master planning in advance but not necessarily real time dispatching. Computation times could be reduced, however, by terminating the IP solver when a feasible solution has been found that is within a specified percentage, say 1%, of the lower bound.

SP used more vehicles than tabu search, however, neither approach included a fixed tanker cost in the objective function. A constraint on the number of tankers could reduce the number of vehicles used at the expense of miles driven and total gas cost.

Table 2 reports the results of the pilot study in the Michigan dispatch area. This area has fewer depots and terminals than the Illinois area. The tabu search approach reduced costs by more than \$53,000 compared to the known GASCO solution over the same five-day period. Additionally, 26 fewer vehicle routes were required. The gap relative to the lower bound ranges from 0.23% to 1.16% suggesting that the solutions are very nearly optimal. Solution times on a 1.3 GHz Athlon PC averaged less than 6 seconds; maximum CPU time was 11 seconds for a Michigan test problem with three depots, 36 tankers, six terminals, and 40 plants to be served on a given day.

SP achieved slightly better results than tabu search on four of the five test problems. The gap for the SP heuristic ranges from 0.21% to 0.43% above the lower bound. On the Michigan test problems an optimal solution could not be achieved. The OSL solver was terminated after 60 hours and the solution reported is the best feasible solution found.

The GASCO solution on Wednesday and Thursday reflect a practice utilized in the Michigan dispatch area called preloading. In preloading, the dispatcher brings a full tanker back to the depot in anticipation of long delivery runs the next day. A preloaded tanker saves both time and mileage for long runs that would otherwise be constrained by the 500 mile travel limit and the 15 hour DOT work limit. The tanker can be loaded by sending it to a terminal and back to the depot or can be preloaded as a backhaul at the end of the preceding route. The preloading strategy constitutes a relaxation of the purchasing routing problem. In Table 2, GASCO utilized preloading on Wednesday with an associated 16.55% increase in cost relative to the lower bound. Purchases were made on Wednesday for Thursday deliveries. Thus the -1.16% gap on Thursday reflects the fact that several purchases were incurred on the previous day. Aggregating the gap on Wednesday and Thursday yields an average gap increase of 7.7% relative to the lower bound. None of our approaches including the lower bound utilizes preloading strategies. Future research will determine the potential benefits of various preloading strategies.

Table 3 shows the performance of the tabu search metaheuristic compared to a savings type heuristic on the three larger test problems. The savings heuristic is based on the heuristic developed by Clarke and Wright (1964) and is often employed as a benchmark in vehicle routing applications [see for example, Golden et al. (1984a,b); Dror and Trudeau (1986); Paessens (1988)]. The savings heuristic had to be modified to handle both the multi-depot and purchasing aspects of the propane gas logistics problem. The initial assignment of plants to depots and terminals to plants was determined by using transportation network flow models.

In generating the results in Table 3, the best of five runs are reported for both the tabu search and savings heuristic. The maximum iterations for the tabu search were 250. From the table, one can see that the CPU times of the tabu search metaheuristic increase nonlinearly with problem size.

Table 2
Results on Michigan dispatch area

Day	GASCO					Tabu search						SP-heuristic solutions ^a					Lower bound Total cost (\$)
	Tanker routes used	Miles driven	Product cost (\$)	Total cost (\$)	Gap (%)	Tanker routes used	Miles driven	Product cost (\$)	Total cost (\$)	Gap (%)	CPU time ^b	Tanker routes used	Miles driven	Product cost (\$)	Total cost (\$)	Gap (%)	
Monday	19	6445	130,731	140,398	10.43	13	4627	120,506	127,446	0.23	0.02	14	4815	120,475	127,698	0.43	127,141
Tuesday	29	10,620	201,239	217,170	1.10	24	8256	203,840	216,224	0.70	0.15	23	8686	202,276	215,305	0.28	214,700
Wednesday	28	10,558	201,698	217,535	16.55	23	8470	175,642	188,348	0.91	0.07	23	8574	174,399	187,259	0.33	186,641
Thursday	27	9754	191,601	206,233	-1.16	22	7740	197,855	209,464	0.38	0.15	21	8167	196,847	209,097	0.21	208,653
Friday	27	9748	176,534	191,156	9.14	22	7684	165,661	177,188	1.16	0.1	21	8140	163,541	175,751	0.34	175,150
Total	130	47,125	901,803	972,492		104	36,777	863,504	918,670		0.49	102	38,382	857,538	915,110		912,285

^a The SP solution process was terminated after 3600 minutes of CPU time on all five problems.

^b CPU minutes using a 1.3 GHz Athlon processor.

Table 3
Comparison of tabu search and savings heuristic

Problem	Tabu search			Savings heuristic			Cost difference (%)
	Tanker routes used	Total cost (\$)	CPU time ^a	Tanker routes used	Total cost (\$)	CPU time ^a	
D 88	33	423,490	1.6	45	426,822	0.004	0.79
D 176	64	848,084	8.7	87	853,654	0.03	0.66
D 264	100	1,275,694	23.9	129	1,285,673	0.09	0.78

^aMinutes on a 1.3 GHz Athlon PC.

The solutions generated by tabu search were slightly lower in gas and mileage cost as compared to the savings heuristic. However, the tabu search approach generated solutions with significantly fewer vehicle routes. For example, the 23 fewer tankers required on problem D176 could yield savings of approximately \$1.9 million per year. Thus, the primary advantage of the tabu search metaheuristic relative to a savings type heuristic is more efficient routing and a significant reduction in vehicle fleet size.

6. Managerial implications

Propane distributors incur several types of costs. Some of these costs pertain to their daily operations and involve purchasing and routing or transportation costs. Other costs involve the location and number of depots, tanker fleet size, and negotiated gas contracts with supply terminals. The solution methodology presented in this paper can provide effective decision support for designing and operating a propane gas supply network. Supply chain networks that consist of seven depots, 25 supply terminals, and up to 264 plants have been solved. The computational results show that the proposed methodology can solve relatively large problems in reasonable CPU times and generate near optimal results.

The primary use of the proposed solution method is in the integration of purchasing and routing decisions made on a daily operational basis. In this study we have shown that significant cost savings can be achieved when compared to manual decision methods as well as a modified savings type routing method. The flexibility of the approach allows the decision maker to gain insight into the tradeoffs and the cost effectiveness of fewer tankers and routes versus more tankers and routes needed to travel extra miles to purchase product at lower prices.

The metaheuristic solution method as well as the SP method can be used in a more strategic application to reconfigure the supply network. They can be used to evaluate the location of depots and the assigned number of tankers and drivers. In this case, the method would need to be applied over an extended period of time such as 1–2 months in order to determine the impact on purchasing and transportation costs. It can also be used to evaluate the allocated capacity at various supply terminals. Some supply terminals have no limit on the number of gallons of propane gas that can be purchased in a given month. Other terminals allocate capacity based on previous usage or negotiated contracts. Using the methodology, the decision maker can evaluate the costs of having more or less capacity at various supply terminals. The methodology can also help to evaluate financial hedging strategies in terms of locking into fixed contracts on future propane gas prices. Key decisions involve how much capacity to reserve in advance at a particular price.

7. Conclusions and future directions

In this paper we have developed a tabu search metaheuristic and a set partitioning model to solve the purchasing/routing problem for propane logistics. Both approaches were effective in solving the purchasing

routing problem. The tabu search metaheuristic generated solutions that were within 1.16% of optimality in both the Illinois and Michigan dispatch areas. Tabu search was also very efficient, generating solutions in less than 10 seconds on a 1.3 GHz PC. It is suitable for use in real time dispatching applications and does not require data preprocessing or expensive optimization software. The SP heuristic was able to generate optimal solutions to two of the Illinois test problems, and solutions that were within 0.43% of optimality for the Michigan test problems. While slightly more robust than the tabu search approach, SP requires orders of magnitude more processing time and is subject to the vagaries of integer programming in which processing time is highly problem dependent and convergence to an optimal solution is not guaranteed. SP is probably more suited to master planning and strategic supply chain configuration analyses.

Compared to actual GASCO routes, both approaches saved more than \$27,000 for the Illinois dispatch area and more than \$53,000 for the Michigan dispatch area during a five-day week. Both tabu search and set partitioning utilized 26 fewer tanker routes (roughly five tankers and drivers per day) for the Michigan dispatch area. Additionally, the tabu search metaheuristic achieved lower costs and approximately a 25% vehicle fleet size reduction when compared to a traditional savings type routing heuristic.

Future research will address the benefits and means of incorporating backhaul and preloading options into the solution method. Further research is also needed to integrate the wholesale inventory/routing policies in conjunction with the retail propane distribution problem.

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