RECONSTRUCTION OF BANDWIDTH-LIMITED DATA ON A SPHERE USING SLEPIAN FUNCTIONS: APPLICATIONS TO CRUSTAL MODELLING

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Models of the crustal magnetic field can be represented using spherical harmonic coefficients. For high-degree models the number of coefficients rapidly becomes large, as a total of $(L+1)^2$ coefficients are required to describe the desired solution complete to a bandwidth of maximum degree L. In addition, as spherical harmonics are a global basis, regions where data are sparse provide poor constraints on the coefficients, which renders the inverse problem ill-conditioned and the solutions poorly behaved locally. In particular, sharp discontinuities are often apparent where regions of differing data density and noise content adjoin. Rather than spherical harmonics, spherical Slepian functions can be employed to produce a locally and also globally orthogonal basis in which to optimally represent the available data in a region at a given degree. The region can have any arbitrary shape and size. The Slepian functions can be tailored to be either band- or space-limited, allowing a trade-off between spectral and spatial concentration in the region and leakage beyond. Another advantage is that only N Slepian coefficients are needed to approximate the signal very well in the region of interest. This "Shannon number", $N = (L+1)^2 f$, where f is the area of the region as a fraction of the area of the entire sphere, effectively reduces the number of free parameters without appreciably degrading the solution. By varying the truncation number of Slepian functions and coefficients in the expansion, the edges of the region can be smoothed, allowing blending between regions of differing data density, for example. We introduce and test this novel methodology using Slepian functions for the reconstruction of crustal field models that are incompletely sampled over selected geographical regions with a variety of bandwidths.

Crustal Modelling; Spherical Harmonics; Slepian functions

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