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Saint-Petersburg State University of Aerospace Instrumentation, St. Petersburg, Russia Service Innovation Research Institute, Helsinki, Finland

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# Some Results for Inhomogeneous $M_t/M_t/S$ Queue with Application to Staffing Problem in Telecommunication Service Systems<sup>\*</sup>

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Abstract. Consideration is given to long-term staffing problem in highlevel telecommunication service systems in which rates of processes that govern their behaviour depend on time. We assume that except for arrivals of requests and their respective service there happen periodic breakdowns when system drops all active requests. The staffing objective is "immediate service of a given percentage of incoming requests". Such service systems typically have strongly time-varying arrivals and services. Noticing that a natural model for such an time-varying processes is an innhomogeneous birth-death process we propose some general theoretical results concerning its ergodicity conditions and limiting behaviour. As an example we show that if service system may be modelled by multiserver queue  $M_t/M_t/S$  with one input flow, periodic arrivals, services and breakdowns (catastrophes) rates which may depend on the state of the system, one can calculate (approximately) the quantities needed for the solution of optimization problem with specified accuracy.

**Keywords:** inhomogeneous birth-and-death process; queueing system; catastrophes; weak ergodicity; bounds on the rate of convergence; limit-ing characteristics.

#### 1 Introduction

The problems of design and management of high-level telecommunication service/information systems remains a topic worthy of research nowadays despite the fact that progress in telecommunication technology during past decade made many things simpler and easier to handle. For analytical study of service/information

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systems, it is common to use stochastic models. There is a big number of research papers devoted to stochastic modelling of service systems in various settings (see, for example, [1], [2] and references therein).

Here we try to contribute into this field by focusing on challenging problem setting in which arrival and service rates are functions of time and there happen periodic system breakdowns with breakdown-rate also dependent on time. The motivation to consider breakdown effects stems from the fact that constantly growing software complexity of systems inevitably leads to situations when the service system becomes unavailable (for example, contact centers of mobile operators) and all currently active requests are dropped by the system. The problem under consideration is the staffing in such service systems with the objective of immediately serving all incoming requests. Specifically, the question is: if one needs to keep, say, more than 80% of incoming requests to be served without waiting, which number of service units allows system to achieve this goal? We assume that no maintenance operations are performed on the system (i.e. it never stops during operation) and its operation period is quite long. The issue concerning uncertainty about the model parameters (arrival, service, breakdown rates) are beyond the scope of the paper and we assume that historical data is enough to estimate them. The specific model we consider is a general multiserver queue  $(M_t|M_t|S$  in Kendall notation) with only one input flow, periodic arrival, service and breakdown (catastrophe) rates which can depend on the state of the queue. This is, of course, a simplified view of the service system. But we believe it gives useful insights into the influence of periodicities on the management of system. The functioning of  $M_t | M_t | S$  queue is described by inhomogeneous birth and death processes with state-dependent additional transitions to the origin (catastrophes). Special cases of inhomogeneous multiserver queueing system  $M_t | M_t | S$  with breakdowns (catastrophes) have been considered [4]. Most recent results for general inhomogeneous birth-and-death models were obtained in [3] and these results allow one to fully describe considered multiserver queue and to calculate (approximately) the quantities needed for optimization problem.

### 2 Example

Here we show how obtained results and be applied to the staffing question. Suppose the service system is modelled by  $M_t/M_t/S$  queue with breakdowns (catastrophes) when intensities are periodic functions of time. Let QoS goal in the service system be to keep more than 80% of incoming requests to be served without waiting. The question is: what number of servers S allows to keep immediate service of incoming requests at a given percentage during long-term operation? Assume that historical data provided the following model parameters. New arrivals of ordinary customers happen according to inhomogeneous Poisson process with intensity  $\lambda(t) = 1000 + 5 \sin 2\pi t$ . Service times follow exponential distribution with intensity  $\mu(t) = 4 + \cos 2\pi t$ . Flow of breakdowns (catastrophes) is Poisson of intensity  $\xi_k(t) = (1 + 1/k)(2 - 2\sin 2\pi t)$  and breakdowns may happen only when system is not empty. Considered queueing system can be described by inhomogeneous birth-and-death process, say, X(t) with state space  $\mathcal{X} = \{0, 1, 2...\}$ . Let  $p_i(t) = \Pr\{X(t) = i\}$  be probability that X(t) is in state i at time t. Denote by  $\mathbf{p}(t) = (p_0(t), p_1(t), ...)^T$  the probability distribution vector at time t. In terms of probabilistic characteristics of the process X(t) the question can be stated as: for which number of servers S the probability  $\sum_{i=0}^{S-1} p_i(t)$  is greater than 0.8? The answer we have obtained is  $S \approx 240$ . The behaviour of considered probability is depicted in Fig. 1.



Fig. 1. Probability of immediately serving all incoming requests on [14, 15].

Our obtained results allow to estimate accuracy. The depicted probability is calculated with error less then  $10^{-6}$ .

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# Non-simulation Solution to Problem of Optimal Routing Strategy of Fixed Size Jobs to n-parallel Servers\*

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Abstract. The routing problems naturally arise in many telecommunication applications including data traffic routing, distributed computation etc. The optimal routing to parallel-working systems is a classical problem to which very few optimality results are known. We try to give some insights into one the general settings of this problem. Specifically consideration is given to a heterogeneous queueing system comprised of n queueing systems working in parallel. Each of n queueing systems consists of one server and infinite queue in front of it. Service times in each queue are deterministic but different. New job upon arrival has to be immediately dispatched into one of n queues where from it is served according to FIFO discipline (no jockeying allowed). The goal is to find optimal policy that minimizes mean sojourn time in the system. Using the fact that for n = 2 the optimal policy is of threshold type, we propose method which does not use statistical simulation techniques and allows one to find value of threshold with desired accuracy. This method can also be applied for the case n > 2 if one uses threshold type policy. But for n > 2 the structure of the optimal policy is unknown and thus the results obtained in such a way may not be optimal.

Keywords: fixed size jobs; job allocation; optimal policy; parallel servers

### 1 Introduction

This note states in short some new results, obtained by authors, which concern the problem of dispatching of fixed-size jobs to several servers working independently in parallel with objective to optimize a certain value function. Consider system at which new jobs of equal fixed size arrive in stochastic manner. Inter-arrival times are independent and have a general distribution. There

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are  $2 \leq n < \infty$  servers in the system, working in parallel and independently of each other. In front of each server there is a queue for jobs of infinite capacity. We assume that servers' speeds are fixed, different and sorted in ascending order i.e. if  $\nu_i$  denotes the speed of server  $i, 1 \leq i \leq n$ , then  $\nu_n > \nu_{n-1} > \cdots > \nu_1 > 0$ . New job upon arrival at the system must be immediately dispatched into one of the n queues and having obtained service leaves the system. No jockeying between queues as well as no service interruptions are allowed. Jobs are served from the queue in first-come, first-served manner. The question of interest is: which policy minimizes mean stationary sojourn time in the system? Consider time instants  $\tau_k$  when k-th job arrives at the system. It is desired to know as a function of the present system state at time  $\tau_k$  into which of n queues k-th job must be dispatched in order to minimize mean stationary sojourn time in the system. The decision maker possesses full information about the state of the system upon arrival of each new job (i.e. number of jobs in each of n queues, residual service time in each of n servers).

It is impossible in such a short note to give more or less full review of problem settings and available results concerning optimal control in distributed systems (optimal job/resource allocation). This is mainly due to active research in this field which is being carried out by many scientific groups and huge number of journal and conference papers which appear regularly. But it should be mentioned that for a class of problems described in the previous paragraph, very few optimality results are known. For a discussion of this fact one can refer to [1] where authors give main results concerning optimal allocation of jobs in system with two parallel servers and Poisson input flow. One of the points of [1] is that although the structure of the optimal policy is known (it is of threshold type), it is not possible to find the exact value of the threshold and one has to apply numerical methods and/or statistical simulation. Moreover, for the case when number of parallel servers is greater than 2 then structure of the optimal policy is unknown and one has find policy for each system state. In what follows we briefly explain the idea which allows one to implement fast numerical algorithm for estimation of optimal threshold value in the case of two parallel servers. It does not rely on statistical simulation and can be generalized to arbitrary number of servers although in such case may not lead to optimal results.

#### 2 Sketch of Idea

It was shown in [1] that the structure of the optimal policy minimizing the value function (mean stationary sojourn time) in the system consisting of two parallel servers (see first paragraph of the previous section) coincides with the structure of the optimal policy in the classical slow-server problem. For the latter from [2, 3] it follows that the optimal policy is of threshold type with only one threshold. The values of optimal thresholds are, of course, different and there is a very simple relation between them (see Lemma 2 in [1]). Nevertheless, the exact expression for the threshold value is unknown. It turns out even impossible to obtain mean stationary sojourn time in the system as a function of the threshold.

This does not allow one to use efficient gradient-based methods in conjunction with statistical simulation for estimation of the threshold value.

Let us denote by  $\xi \geq 0$  the unknown value of the optimal threshold. Our idea to estimate  $\xi$  is based on the following fact: if  $\xi$  is indeed the value of the optimal threshold, then the alternative (dispatch to server 1 or server 2), which is picked at time instant, say  $\tau_0$ , when system's state lies on the border (defined clearly by the value  $\xi$ ), must not have influence on the value function. Choose arbitrary value  $\xi_0 \geq 0$  and suppose that at time  $\tau_0$  system's state lies on the border (for example, the residual service time in server 2 is  $\xi_0$ , server 1 and both queues are empty) and new job arrives at the system. Introduce quantities  $g_k^{(i)}$ , i = 1, 2, - mean sojourn time in the system at time instant  $\tau_k$ ,  $k \geq 0$  if at time instant  $\tau_0$  the decision was to dispatch job to server *i*. Then the sign of the sum

$$\sum_{k=0}^{\infty} (g_k^{(1)} - g_k^{(2)}),$$

which must be equal to zero if  $\xi_0 = \xi$ , shows how close the initially picked value of threshold  $\xi_0$  is close to the value of the optimal threshold  $\xi$ . If the sum is greater then zero the the next test value of threshold must be lower than  $\xi_0$  and otherwise it must be greater. This iterative procedure leads to determination of  $\xi$  with needed accuracy. For calculation of  $g_k^{(i)}$  one needs to introduce constrains into system's state-space which coincides with  $\mathbf{R}^2_+$  and perform its discretization.

Our numerical experiments show, that the sum of  $(g_k^{(1)} - g_k^{(2)})$  is very sensitive to the quality of discretization grid. Our best results are achieved using non-uniform grid, when discretization of  $\mathbf{R}^2_+$  is done using lines with slope  $\nu_2/\nu_1$ .

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# On an Adaptive Data Prediction in the Information Systems<sup>\*</sup>

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**Abstract.** The paper describes some prediction models which may be used in the real information systems, aimed for a data storage, transmission and a design of the data mining systems. The models are "adaptive" in a sense of an absences of the fundamental assumptions about the probability distributions of the sample. We present a special software tool based on the method of moving separation of the finite probability mixtures to ascertain a possibility of a forecasting for the data.

**Keywords:** prediction algorithms, moving separation of mixtures, finite mixtures of probability distributions, data mining

### 1 Introduction

A prediction of a sequential data is still a fundamental task for a network traffic control (in particular, the effects of attacks prediction), a data transition and storage, an electric power load forecasting, a financial markets analysis, etc. The applications involving sequential data may require a prediction of new events, a generation of new sequences or a decision making such as classification of sequences or sub-sequences. It is well known that the prediction is tightly connected with lossless compression because any compression is based on information about the future values of data compressed by some previous values.

The paper describes some prediction models which may be used in the real information systems, aimed for a data storage, transmission and a design of the data mining systems. The models are "adaptive" in a sense of an absences of

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the fundamental assumptions about the probability distributions of the sample. We present a special software tool based on the method of moving separation of the finite probability mixtures to ascertain a possibility of a forecasting for the data.

We describe some methodologies and software solutions for data prediction. The tool can be used for an analysis of the data predictability taking into account their stochastic nature at a system design stage.

### 2 On a Prediction within an Unknown Data Probability Distribution

We consider the problem of a prediction for the average value  $\mathbb{E}(X_n|X_0,\ldots,X_{n-1})$  of a random sequence at time t, given the values  $X_0,\ldots,X_{n-1}$ , i.e., in the case where statistical characteristics of the process are a priori unknown [1], [2]. However, in a contrast to the numerous approaches, we do not assume the type of distribution )(not only its parameters, but any Markovian properties of the data source too). Formally, we should predict, or estimate, the occurrence probability of a symbol d from a finite alphabet D for an element of the sequence with index n + 1(corresponding to time t + 1) that is

$$\mathbb{P}(X_{n+1} = d | X_1 = x_1, X_2 = x_2 \dots, X_n = x_n).$$

Some asymptotic algorithms of a prediction can be based on the well-known universal Lempel-Ziv (LZ77,LZ78) algorithms for a random sequences entropy estimation [3]. An LZ78-based prediction algorithm was proposed in [4]. This algorithm allows to compute the conditional probability mentioned above. However, the L78 algorithm is only a universal prediction algorithm with respect to the large class of the stationary and ergodic Markov sources of a finite order. It provides good compression/prediction in an asymptotic sense, but for the real data it requires some additional considerations. In spite of our good previous results with this approach [5] we suggest a way for a prediction using another adaptive approach to the probability estimation, which is based on a model of a finite mixture of probability distributions and a method of a moving separation of mixtures (MSM method) [6].

### 3 An Adaptive Tool for a Data Prediction

Instead of a hypothesis about a probability distribution of the data we use enough general assumption about the model of an approximating distribution. Suppose  $\mathcal{F}(x)$  is a finite mixture of two-parameter probability distributions with unknown parameters, i.e.:

$$\mathcal{F}(x) = \sum_{i=1}^{k} p_i F(x, a_i, b_i), \text{ where } \sum_{i=1}^{k} p_i = 1, \ p_i \ge 0,$$
(1)



Fig. 1. The data and increments within the histograms.

 $F(\cdot)$  denotes some type of cumulative distribution functions;  $k \ge 1$  is a known natural number;  $a_i, b_i, i = 1, ..., k$ , are parameters of distribution under correct conditions. Quantities  $p_1, ..., p_k$  are called "weights"; k is a number of components in mixture. Values  $p_i, a_i, b_i, i = 1, ..., k$ , are usually unknown and should be estimated by sample.

Let consider the main stages of analysis within a special software tool based on the MSM method. First of all, we are interested in a character of the initial data. The typical data for analysis are shown by Fig. 1, the upper left graph. It is an example of a some index of a company effectiveness.

There is a strong non-stationarity of the data, so the increments of data should be analyzed in order to be able to predict a next  $X_{t+1}$  value given previous  $X_1, \ldots, X_t$ . The upper right graph on Fig. 1 demonstrates the curve for increments. Further, the lower graphs on Fig. 1 represents the approximating distributions of the data and their increments by the finite normal mixtures.

The next step is an application of the MSM method for the increments [6]. The initial data (increments) is divided into the moving sub-samples with a same size ("window"). The width of window is a parameter of the MSM method. At each step of method the window is slid by adding of a one new element from initial data to the end of window. The first element of a previous window is excluded from the current analysis. So, we can see a time evolution of changes in a data structure.

Thus, we may estimate the data distribution without any preliminary assumptions about their nature, then we can use these distributions for a computation of the conditional expectation mentioned above (for example, as in [7]).

The essential changes in a data structure is a very important index of the non-stationarity and this is a factor preventing for a quality forecasting. We may indicate such data domains by so-called *dynamic component* of the volatility of the process, which is described by the parameters  $a_i$  from relationship (1) (see Fig. 2).

The color of each component is determined by a set from deep blue for weights which are close to 0 to deep red for weights which are close to 1 (it



Fig. 2. A dynamic component of the volatility. EM algorithm, 2 components in a mixture, length of window equals 100.

is demonstrated by colorbar at the right side of a figure). The x-axis shows a position of window (its number).

Note that the data on Fig. 1 can be digitalized in order to be predicted with the lossless compression-like algorithms mentioned above.

### 4 Conclusions

In this paper we study some opportunities of a data prediction in the case of absent knowledge about their probability distributions. The data are the financial indexes, the network attacks sequences. We show that this prediction can be supported with some software tools, which are based on the model of a finite mixture of probability distributions and the MSM method.

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# On a Solution for an Investigation of the Information Flows Structure<sup>\*</sup>

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Abstract. The paper describes an approach to development of an architecture of software analysis of information flows within the mixture models of probability distributions. The distributed computing can be used for a large data processing which can be obtained from different areas.

**Keywords:** data mining, probabilistic models, online computing, big data, information technology

### 1 Introduction

The methods and approaches for the construction of structural models for the information flows are very diverse. Let us consider some examples of papers in which stochastic models are used for the various kinds of information systems.

The paper [1] describes a creation of an information flows model based on Markov chains with a help of the method of Monte Carlo simulation (due to a significant computational complexity of the probability estimators). The paper [2] demonstrates an analysis of the stock index S&P500 based on the symmetric scale mixtures of normal distributions. In particular, it allows to work with the variance-gamma distribution. In [3] the Australian dollar exchange rate is investigated with a representation of the student's t-distribution within a scale normal mixture. Also, the software tool WinBUGS is suggested for a computation.

The models based on the various probability distributions are very popular for the application in the information systems. The software realization of the models is very important due to the problems of data processing.

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The WolframAlpha project (http://www.wolframalpha.com/) should be mentioned as a potential analogue but even a classical EM algorithm [4] is realised in WolframAlpha as a package for Mathematica and it is not available online. The same situation stands for the more complicated algorithms too.

In this paper we describe the structure of an online realization of a solution based on the probabilistic models.

### 2 Mathematical Models

As the theoretical background we use the compound Cox processes and mixture distributions (see the book [6]). In practice, during the statistical analysis based on the model of compound Cox processes an approximation by the finite and continuous mixtures of distributions is used.

The first type often allows us to specify the precise formulas for estimators of unknown parameters (for example, in a case of finite normal mixtures [6]). This model is also very convenient for the practical interpretation by matching the mixture components (it is more correctly to talk about the components of a process volatility) with the real-life processes.

The main disadvantage of this type of models is a discrete essence of a mixing distribution. In practice, we have to solve the problem with an arbitrary number of unknown parameters because the parameter k is often indeterminate too. Moreover, under determined value of k, the 3k - 1 parameters should be estimated. The models for real processes are mostly based on 3 - 6 mixture components so it implies 8 - 17 estimators. It may be a serious problem for the real-time (or similar) systems.

The models based on the continuous mixtures are more complicated but we can use a fixed number of parameters keeping necessary probabilistic properties for various samples. One of the most interesting types of such distribution is a normal variance-mean mixture (see, for example, [5]). The computational procedure for finding of estimators is non-trivial in this case, the possibility of the results interpretation is not obvious. But the universalism of such models is a very important.

### 3 Architecture of Solution

During the work we can artificially divide the project into several semi-incapsulated parts which are shown on Fig. 1.

First is the client workstation and HTML/JS client that the user works with. It provides the user with a basic interface, allowing for information input and the usual variety of actions. It also serves as a graphic representation tool that constructs data visualisation from the JSON objects sent by server. Currently it is supposed that either RaphaelJS or AnyChart solutions will be applied in the project for both of them are capable of dealing with sufficiently large data sets.

Frontend Server is responsible for formation of the HTML/JS templates and pages, information validation and different miscellaneous tasks. Client workstation communicates only with frontend server and it processes most of the logic that is not connected with calculations. Login/logout attempts, account and project management, file uploads and downloads all are processed (or at least validated) on this server.



Fig. 1. The general architecture.

Backend Server works as a connectivity link between the Frontend Server and Database & Job Servers. It gets data from SQL Database and outputs it to Frontend Server, it runs various asynchronous daemons and it also forms tasks for Job Servers. Once the user had entered the data set for the count, it's the Backend Server which will divide it into subsets and send to Job Servers for calculations.

Database Server and possible Slave Database Server are responsible for running and replicating database, including backups. Currently it is supposed that no replication will be needed but it can be added in future to lower the server load and avoid possible delays due to table locks.

Job Servers receive tasks from Backend Server, process them and respond with model parameters. All difficult calculations, including gamma functions and integration are processed there. Furthermore, they keep cache tables for such calculations.

#### 4 Conclusions

The realization of the ideas, mentioned above, could be useful for many practice areas. The successful results for some of the described probabilistic models are well known (for example, applications for financial markets, plasma turbulence physics, etc. [7]). So, the implementation of the software solution based on the suggested methodology seems to be very perspective.

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# Stochastic Approach to Protection of Network Resources

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**Abstract.** The problem of acceleration of decision-making by system of intrusion detection is considered. It is offered to protect valuable resources, using appearance of security events in process of their generation. The probabilistic model of such protection is constructed. The estimates received in this model validate the approach.

**Keywords:** intrusion detection; decision-making by system; probabilistic model;

### 1 Introduction

Systems of information security monitoring and intrusion detection are essential part of computer security systems, including distributed computer systems. An intrusion detection is provided with two approaches [1]:

- recognition of attacks by means of signatures or rules;
- detection and the analysis of anomalies in information processes.

Each of methods possesses the pluses and minuses.

The paper presents the solution of the problem of stability of attacks recognition and proposes an expansion of signature methods of detection for identification of new attacks. Transition to a filtration of sets of potentially attacked objects is the cornerstone of a method. The idea of approach arose at research of bans of probability measures [3, 4].

### 2 Mathematical Model and Basic Results

Instead of intrusion detection it is more convenient to speak about damage prevention. The System of Damage Prevention (SDP) is based on timely identification of the network attacks directed on assets of information system.

The information resources, the software, technological decisions realized in the form of sequence of procedures, knots of the distributed systems, etc. can be assets. Creation of any protection begins with accurate definition of assets. Let  $A_1, ..., A_R$ , be the protected assets and be the purposes of attacks.

However attacks in modern systems can't be realized in one step. Each attack represents sequence of the actions in computer system allowing to get an access to object of attack and to make damage.

In order that SDP could prevent attacks it is necessary to collect information on the existing attacks and ways of their realization. Formally in the solution of tasks by means of SDP there are two stages:

- The first stage consists in training of system on the basis of the analysis of trees of attacks and the available examples of the attacks generating sequences of security events. The choice of the traced events is an independent task and therefore in this article isn't considered.
- The second stage consists in presentation to SDP of data from sensors of security events, and making decision, whether there is an attack, what are the assets which are attacked, etc. This problem is solved with the help of metrics which not necessarily define the fact of existence of attack, and SDP gives the prevention to the administrator that there are signs of attack to valuable assets.

Data when SDP is being trained can be presented in the following form. We will denote examples of attacks through  $X = \{o_1, ..., o_m\}$ . In systems without semantics use each attack is described by a set of events (which also we will call properties), which the set of sensors transfers to SDP. The set of all properties which the system can observe is designated through  $Y = \{\alpha_1, ..., \alpha_n\}$ . Then the training information can be presented in the matrix form  $M_{m \times n}$ , where on a place (i, j) costs 1 if *i*-th attack generates *j*-th event (or possesses property  $\alpha_j$ ). All other elements of a matrix are equal to 0.

For simplicity we will consider that each attack is aimed only at one asset. Then due to collected examples of attacks a function F, which maps a set X on set of assets  $A_1, ..., A_R$ , is defined. It is obvious that function F defines on X the relation of equivalence. That is  $o_i \sim o_j$  in only case when  $F(o_i) = F(o_j)$ .

SDP works with the data obtained by information system on one or several channels. By means of technology of parallelization an entrance data are analyzing in parallel with receiving of these data in information system for processing. In parallel the working system of monitoring sends the message about the received properties from a set Y to the center of the analysis. The analysis is based on identification of the class of equivalence generated by function F, and turning on of additional mechanisms for protection of the attacked asset. An example

of such reaction of the system is the blocking of access to assets in a case when there is a confidence that attack is carried out.

In the paper it is offered to carry out a consecutive exception of assets of the possible purposes of attacks and therefore not to allow the blocking of access to all assets at once.

To accurately determine algorithm of an assets exception from among the attacked assets by observed signs, we will define a new matrix  $K_R \times n$  as follows. Lines of a matrix of K are numbered by assets  $A_1, ..., A_R$ , columns of matrix K are numbered by set of elements  $Y = \{\alpha_1, ..., \alpha_n\}$ . The unit on a place  $(A_i, \alpha_j)$  is put in only case when the submatrix of matrix M generated by an equivalence class  $F^{-1}(A_i)$  contains at least one unit in a column  $\alpha_j$ . In this case we'll call the property  $\alpha_j$  by a ban for the asset  $A_i$ . If in a matrix K there is 0 in  $(A_i, \alpha_j)$ , it means that property  $\alpha_j$  didn't appear in attacks on  $A_i$ .

The offered algorithm keeps the working ability of information system during attack and it works as follows. Let  $\alpha_{i_1}, \alpha_{i_2}, ...$  be the sequence of the signs observed by sensors. At emergence  $\alpha_{i_1}$  all assets  $A(\alpha_{i_1}) \subseteq A$  are excluded, where  $A(\alpha_{i_1})$  is defined by the ban  $\alpha_{i_1}$ . At emergence of sign  $\alpha_{i_2}$  all assets  $A(\alpha_{i_1}, \alpha_{i_2}) \subseteq A(\alpha_{i_1})$ , which are defined by bans signs  $(\alpha_{i_1}, \alpha_{i_2})$  are excluded, etc. All assets which don't include on step k to the set  $A(\alpha_{i_1}, \alpha_{i_1}, ..., \alpha_{i_k})$  can be used without restriction since observed attack doesn't concern them.

We will assume that time T during which attack to the asset can be fulfilled is known. For simplicity we will consider that for all assets and for all attacks a time T is the same. Then owing to a lag effect of monitoring system a concrete attack can be not revealed, but attack can happen. To avoid such situation in a time T it is necessary to block all assets from a set  $A(\alpha_{i_1}, ..., \alpha_{i_k})$ , which is generated by signs of attacks revealed by the time T. Thus other assets can freely be used.

This method of preservation of workability of information system has a stochastic character. In an elementary model we will estimate quantity of assets which can be used, despite attack. As we will be interested in temporary characteristics of identification of attacks, we will consider that the random variable equal to time of identification of an event, delivery of an event in SDP, and preprocessing of this event in SDP is connected with each unit of a matrix K. As all temporary characteristics in processing of security events are independent and proceed quickly, it is possible to assume that distribution of these random variables submit to the exponential law with the same parameter  $\lambda$ . Then the number of security events during T approximately has Poisson's distribution with parameter  $\lambda T$ .

Let's consider one more simplifying assumption: independent placements with probability p of units in matrix K. Then after the observation of k security events the probability that an asset doesn't belong to a set of potentially attacked assets equals to  $(1-p)^k$ . Then average of the available to use assets in time T equals to

$$E(T, p, \lambda) = R \exp^{-\lambda T p}.$$
(1)

### 3 Complexity of Assets Isolation Algorithm

When the matrix K is constructed, complexity of formation of a set of the forbidden assets at emergence of k security events is estimated by size Rk + nk. From here, using an assumption about Poisson distribution of security events during T, we receive an average assessment of complexity  $(R + n)\lambda T$ .

These estimates show the efficiency of high-speed characteristics of the method offered in the paper.

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# Accelerated Verification of Stability Condition of Multiserver System with Simultaneous Service \*

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**Abstract.** An earlier found stability criterion of a multiserver model with simultaneous service is numerically consuming, and we propose a new approach which allows to accelerate verification of the stability condition for large number of servers. This approach reduces a state space related to calculation of a basic normalization constant included in the stability condition. Moreover, this result allows to obtain a stable configuration of the supercomputers with a given input rate.

**Keywords:** multiserver system, stability condition, normalization constant, search reduction, green computing

#### 1 Introduction

The increasing interest to multiserver systems with simultaneous service and, in particular, to their stability, is motivated by the modeling of high performance clusters (HPC) and cloud/distributed computing containing a huge number of servers working in parallel. The key feature of systems with simultaneous service is that there is a possibility to have the idle servers and a non-empty queue simultaneously (if a customer requires more servers than are available at the moment). In other words, the service discipline in such systems is not workconserving, and it dramatically complicates the stability analysis. The knowledge of the exact stability condition of the model is of an important practical interest. For instance, it allows to select an upper bound for the computational time of a task at an HPC (say, the so-called runtime estimate in a queue manager SLURM) [3]. Moreover, the knowledge of stability condition allows to use the DVFS technology [7] to lower the CPU frequency and save energy. In the recent paper [3], stability criteria of such systems (under exponential assumptions) has been obtained in an explicit form. This explicit expression contains a normalization constant which, when the number of servers is large, is difficult to be

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calculated. By this reason, a reduction of the corresponding sorting (being the main part of the calculation) becomes crucially important to make calculation of the constant available in practice. In this work we suggest such an acceleration method.

### 2 An Explicit Formula for Normalization Constant in Stability Criteria

Consider a FCFS s-server (CPUs) system with Poisson input with arrival epochs  $\{t_i, i \ge 1\}$  with (exponential) interarrival times  $T_i := t_{i+1} - t_i > 0$  and rate  $\lambda = 1/ET$  (*T* is generic interarrival time). Each customer *i* requires  $N_i$  servers simultaneously for an exponential service time  $S_i$  with rate  $\mu = 1/ES_i$ . (We call it class- $N_i$  customer.) Provided the number of the available servers is not enough (head-of-line) customer *i* must wait in the queue until the required number of the servers become available. The random variables  $\{N_i\}$  are i.i.d. with given distribution  $\mathbf{p} = (p_1, \ldots, p_s)$ ,

$$p_k := \mathsf{P}\{N = k\}, \quad k = 1, \dots, s,$$
 (1)

where N is a generic variable. Let  $\nu(t)$  be the number of customers in the system at time t. Define a state of the system (vector)  $m(t) = (m_1(t), \ldots, m_s(t))$ , where  $m_i(t) \ge 0$  is the number of the servers required by the *i*th oldest customer present in the system at time  $t^-$  ( $m_i := 0$ , if there is no such a customer). Let  $\mathcal{M} = \{1, \ldots, s\}^s$  be the set of the states. It is shown in [3] that

$$\{X(t) := (\nu(t), m(t)), t \ge 0\} \in \mathbb{Z}^+ \times \mathcal{M}$$

is a Quasi-Birth-Death (QBD) process with the state space  $\mathcal{M}$ . The stability criteria of the process  $X(t), t \ge 0$ , is [3]

$$\rho := \frac{\lambda}{\mu} C(s) < 1, \tag{2}$$

where the normalization constant

$$C(s) := \sum_{m \in \mathcal{M}} \frac{1}{\sigma(m)} \prod_{i=1}^{s} p_{m_i}, \qquad (3)$$

and  $\sigma(m) := \max\{i : \sum_{j=1}^{i} m_j \leq s\}$  is the number of the customers being served at the state m.

In general (when the number of servers s is large) a direct computation of C(s) via (3) is dramatically time-consuming. Instead, the following equivalent and computationally effective representation has been presented in [1]:

$$C(s) = \sum_{k=1}^{s} \sum_{n \in \mathcal{N}_k} \frac{(\sum_{i=1}^{s} n_i - 1)!}{\prod_{i=1}^{s} n_i!} \prod_{i=1}^{s} p_i^{n_i} \sum_{j=s-k+1}^{s} p_j,$$
(4)

where  $n \in \mathcal{N}_k$  is an integer partition of the number  $k = 1, \ldots, s$  in the form  $(1^{n_1} \ldots s^{n_s})$ , i. e.,  $k = n_1 + 2n_2 + \cdots + sn_s$ . However equation (4) requires computation efforts asymptotically (as  $s \to \infty$ ) of the order  $e^{\sqrt{s}}/s$ . Although it is much better than the exhaustive search in (3) (having order  $s^s$ ) it is still computationally consuming.

In this work, we suggest a further acceleration of the calculation of the constant C(s). The idea is to replace formula (4) based on an exhaustive search over the number of busy servers k, by the search over the number of customers at service  $\sigma(m) = 1, \ldots, s$ . Note that the latter quantity is typically much less than the former one.

For some  $k \ge 1$ , we define the subset of  $\mathcal{M}$  such that there are exactly k customers at service:

$$\mathcal{M}^{(k)} = \{ m \in \mathcal{M} : \sigma(m) = k \}.$$
(5)

Then (3) gives

$$C(s) = \sum_{k=1}^{s} \frac{1}{k} \sum_{m \in \mathcal{M}^{(k)}} \prod_{i=1}^{s} p_{m_i}.$$
 (6)

Since, for any  $m \in \mathcal{M}^{(k)}$ , the (k+1)th oldest customer is waiting in the queue, then  $m_{k+1} \ge s+1-\sum_{i=1}^{k} m_i$ . The (k+i)th oldest customer, for each i > 1 (if any), may require an arbitrary number of servers, and this requirement is independent of the components  $1, \ldots, k+1$  of vector m. Thus,

$$\sum_{m \in \mathcal{M}^{(k)}} \prod_{i=1}^{s} p_{m_i} = \sum_{j=k}^{s} \sum_{m \in \mathcal{F}(k,j)} \prod_{i=1}^{k} p_{m_i} \sum_{t=s+1-j}^{s} p_t,$$
(7)

where  $\mathcal{M}^{(k,j)} := \{m \in \mathcal{M}^{(k)} : \sum_{i=1}^{k} m_i = j \ge k\}$ . It remains to note that  $\sum_{m \in \mathcal{M}^{(k,j)}} \prod_{i=1}^{k} p_{m_i}$  is exactly the *j*-th component of a *k*-fold convolution  $p_j^{k*}$  of the distribution  $\{p_k\}$ , which is iteratively defined as

$$p_j^{2*} = \sum_{i=1}^{j-1} p_i p_{j-i}.$$
(8)

Finally, it follows from (5)-(8) that the following statement holds.

**Lemma 1.** The value C(s) in (2) satisfies

$$C(s) = \sum_{k=1}^{s} \frac{1}{k} \sum_{j=k}^{s} p_j^{k*} \sum_{t=s+1-j}^{s} p_t.$$
(9)

Note that  $p_j^{k*}$  is the probability of the event  $\{N_1 + \cdots + N_k = j\}$ , where  $\{N_i\}$  are the i.i.d. copies of the generic r.v. N. Thus, the algorithm for accelerated calculation of the constant C(s) can be written as the following R-language [4] code (which is now a part of the hpcwld package [5,2])

```
A=cumsum(p[s:1])
B=p
C=0
for(i in 1:depth){
C=C+sum(B[1:(s-i+1)]*A[i:s])/i
B=convolve(B,rev(p),type="o")
}
C
```

The convolution operation may be easily performed by means of Fast Fourier Transform [6], and it gives an additional gain in calculation.

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# On Vertex Degrees in a Conditional Configuration Graph \*

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Abstract. We consider a configuration random graph with N vertices whose degrees are independent and identically distributed according to generalized power-law distribution. Studies carried out in the past decades showed that such random graphs are deemed to be a good models of complex networks, e.g. Internet. The asymptotic structure of the configuration graph largely depends on the vertex degrees. As  $N \to \infty$  the limit theorems for maximum vertex degree and number of vertices of a given degree were proved.

**Keywords:** configuration graph, generalized power-law distribution, maximum vertex degree, vertex of a given degree.

Interest in the study of random graphs has been growing in connection with the description of complex networks, for instance the World Wide Web (see e.g. [1]–[3]). Observations of real networks [1] showed that their topology could be described by random graphs with vertex degrees represented by independent identically distributed random variables. In [3] it was suggested using models based on configuration random graphs introduced in [4]. It is known [3] that for large k the number of vertices with degree k is proportional to  $k^{-\tau}$ , where  $\tau > 0$ . Hence, we can assume that the distribution of vertex degree  $\eta$  is

$$\mathbf{P}\{\eta \ge k\} = h(k)k^{-\tau+1}, \qquad k = 1, 2, \dots,$$
(1)

where h(k) is a slowly varying function. We consider a random graph consisting of N + 1 vertices. The random variables  $\eta_1, \ldots, \eta_N$  are equal to the degrees of vertices with the numbers  $1, \ldots, N$ . Each vertex is given a certain degree in accordance with the degree distribution (1). The vertex degree is the number of stubs (or semiedges) that are numbered in an arbitrary order. Stubs are graph edges for which adjacent nodes are not yet determined. The vertex 0 has the degree 0 if the sum of degrees of all other vertices is even, else the degree is 1. The graph is constructed by joing all the stubs pairwise equiprobably to form edges.

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We consider the subset of random graphs under the condition that  $\eta_1 + \ldots + \eta_N = n$ . Analysis of conditional random graphs was first carried out in [5]. Here we extend these results to the configuration graphs with degree distribution (1). Let

$$p_k = \mathbf{P}\{\eta_1 = k\} = \frac{h(k)}{k^{\tau} \Sigma(1, \tau)},$$

where  $k = 1, 2, ..., \tau > 0$ , h(x) is a slowly varying function integrable in any finite interval and

$$\Sigma(x,y) = \sum_{k=1}^{\infty} x^k \frac{h(k)}{k^y}.$$

We denote by  $\xi_1, \ldots, \xi_N$  auxiliary independent identically distributed random variables such that

$$p_k(\lambda) = \mathbf{P}\{\xi_i = k\} = \frac{\lambda^k p_k \Sigma(1, \tau))}{\Sigma(\lambda, \tau)},$$
  
$$i = 1, 2, \dots, N, \qquad k = 1, 2, \dots, \quad 0 < \lambda < 1.$$

Let  $\lambda = \lambda(N, n)$  be determined by the relation

$$m = \mathbf{E}\xi_1 = \frac{\Sigma(\lambda, \tau - 1)}{\Sigma(\lambda, \tau)} = \frac{n}{N}.$$

Denote by  $\eta_{(N)}$  and  $\mu_r$  the maximum vertex degree and the number of vertices with degree r, respectively. The next theorems are valid.

**Theorem 1.** Let  $N, n \to \infty, 1 < C_1 \leq n/N \leq C_2 < \Sigma(1, \tau - 1)/\Sigma(1, \tau), r = r(N, n)$  take values in such a way that

$$\frac{N\lambda^{r+1}h(r+1)}{(r+1)^{\tau}\Sigma(\lambda,\tau)} \to \gamma,$$

where  $\gamma$  is a positive constant. Then for any fixed  $k = 0, \pm 1, \ldots$ 

$$\mathbf{P}\{\eta_{(N)} \le r+k\} = \exp\left\{-\frac{\gamma\lambda^k}{1-\lambda}\right\} (1+o(1)).$$

Let

$$\sigma_{rr}^2 = p_r(\lambda) \left( 1 - p_r(\lambda) - \frac{(m-r)^2}{\sigma^2} p_r(\lambda) \right),$$

where

$$\sigma^{2} = \mathbf{D}\xi_{1} = \frac{\Sigma(\lambda, \tau - 2)}{\Sigma(\lambda, \tau)} - m^{2}.$$

**Theorem 2.** Let  $N, n \to \infty, 1 < C_1 \leq n/N \leq C_2 < \Sigma(1, \tau - 1)/\Sigma(1, \tau), r$  is a fixed positive integer. Then

$$\mathbf{P}\{\mu_r = k\} = \frac{1 + o(1)}{\sigma_{rr}\sqrt{2\pi N}} \exp\left\{-\frac{u_r^2}{2}\right\}$$

uniformly in the integers k such that  $u_r = (k - Np_r(\lambda))/(\sigma_{rr}\sqrt{N})$  lies in any finite fixed interval.

**Theorem 3.** Let  $N, n, r \to \infty, 1 < C_1 \leq n/N \leq C_2 < \Sigma(1, \tau - 1)/\Sigma(1, \tau)$ . Then

$$\mathbf{P}\{\mu_r = k\} = \frac{1+o(1)}{k!} (Np_r(\lambda))^k \exp\left\{-Np_r(\lambda)\right\}$$

uniformly in the integers k such that  $(k - Np_r(\lambda))/\sqrt{Np_r(\lambda)}$  lies in any finite fixed interval.

The technique of obtaining these theorems is based on the generalized allocation scheme suggested by V.F.Kolchin [6]. It is readily seen that for our subset of graphs

$$\mathbf{P}\{\eta_1 = k_1, \dots, \eta_N = k_N\} = \mathbf{P}\{\xi_1 = k_1, \dots, \xi_N = k_N | \xi_1 + \dots + \xi_N = n\}.$$

Therefore the conditions of the generalized allocation scheme are valid.

Let  $\xi_1^{(r)}, \ldots, \xi_N^{(r)}$  and  $\tilde{\xi}_1^{(r)}, \ldots, \tilde{\xi}_N^{(r)}$  be two sets of independent identically distributed random variables such that

$$\mathbf{P}\{\xi_1^{(r)} = k\} = \mathbf{P}\{\xi_1 = k | \xi_1 \le r\},\$$

$$\mathbf{P}\{\xi_1^{(r)} = k\} = \mathbf{P}\{\xi_1 = k | \xi_1 \neq r\}, k = 1, 2, \dots$$

We also put  $\zeta_N = \xi_1 + \ldots + \xi_N$ ,  $\zeta_N^{(r)} = \xi_1^{(r)} + \ldots + \xi_N^{(r)}$ ,  $\tilde{\zeta}_N^{(r)} = \tilde{\xi}_1^{(r)} + \ldots + \tilde{\xi}_N^{(r)}$ ,  $P_r = \mathbf{P}\{\xi_1 > r\}$ . It is shown in [6] that

$$\mathbf{P}\{\eta_{(N)} \le r\} = (1 - P_r)^N \frac{\mathbf{P}\{\zeta_N^{(r)} = n\}}{\mathbf{P}\{\zeta_N = n\}},$$
(2)

$$\mathbf{P}\{\mu_r = k\} = \binom{N}{k} p_r^k(\lambda) (1 - p_r(\lambda))^{N-k} \frac{\mathbf{P}\{\tilde{\zeta}_{N-k}^{(r)} = n - kr\}}{\mathbf{P}\{\zeta_N = n\}}.$$
(3)

From (2) and (3) we see that to obtain the limit distributions of  $\mu_r$  it suffies to consider the asymptotic behaviour of the binomial  $(1 - P_r)^N$ , the binomial probabilities

$$\binom{N}{k} p_r^k(\lambda) (1 - p_r(\lambda))^{N-k}$$

and sums of auxiliary independent identically distributed random variables  $\zeta_N$ ,  $\zeta_N^{(r)}$ ,  $\tilde{\zeta}_N^{(r)}$ . To solve these problems one has to find both integral and local convergence of the distributions of these sums to limit laws under the conditions of array schemes, which are the main difficulties.

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### Forest Fire Modeling on Configuration Graphs<sup>\*</sup>

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**Abstract.** We consider configuration graphs with independent identically distributed node degrees drawn from either power-law or Poisson distribution. By computer simulations we study a process of fire propagation over the graph links, considering two ways of fire ignition: random and targeted. For both graph types we estimated the optimal values of the node degree distribution parameters that ensure maximum node survival in case of a fire. We compared the two configuration graph models of the same sizes to specify a "better" one in terms of the number of nodes surviving in the fire.

**Keywords:** configuration graph, power-law distribution, Poisson distribution, robustness, forest fire model, simulation modeling.

Configuration graph models have recently become one of the interesting and attractive objects for both theoretical and experimental studies due to the wide use of these models for the representation of various types of complex networks (see e.g. [1]). Among the different research trends in the field of random graphs the study of their robustness has always remained one of the most important (see e.g. [2,3]). Theoretical approaches do not however always outpace experimental research. Thus, we have lately been actively using simulation modeling for studying configuration graphs resilience to different types of breakdowns [4– 6]. We have been considering two types of graph destruction process. The first approach deals with preserving graph connectivity [4] and, the second one refers to the question of node survival which arises from an issue related to forest fire modeling [4–6]. The second approach could be used to model both forest fires [7] and banking system defaults to minimize their negative effects [8].

We consider configuration graphs introduced in [9] with node degrees being independent identically distributed random variables following one of the two distributions: power-law (1) or Poisson distribution with a single shift (2):

$$\mathbf{P}\{\xi \ge k\} = k^{-\tau}, \qquad k = 1, 2, \dots, \ \tau > 1, \tag{1}$$

$$\mathbf{P}\{\xi = k+1\} = \frac{\lambda^k}{k!} e^{-\lambda}, \qquad k = 0, 1, \dots, \ \lambda > 0.$$
(2)

 $<sup>^{\</sup>star}$  The study was supported by the Russian Foundation for Basic Research, grant 13–01–00009.

As the distribution (1) or (2) defines the number of stubs for each graph node, all the stubs are joined one to another equiprobably to form links. If the sum of node degrees is odd one stub is added to a random node to form a lacking link.

To associate configuration graph model with a confined area of a real forest we used auxiliary graphs of the topology represented by a square lattice sized  $100 \times 100$ . The link between nodes exists if the fire can pass from one node to another. We used these auxiliary graphs to estimate the dependence between the parameters of node degree distributions ( $\tau$  or  $\lambda$ ) and the graph size N:

$$N = [9256\tau^{-1.05}], \qquad R^2 = 0.97, \tag{3}$$

$$N = [907.5\lambda + 2509.4], \qquad R^2 = 0.98, \tag{4}$$

where  $R^2$  is the determination coefficient.

We considered two possibilities of starting a fire: a random ignition when the first node to be set on fire is chosen equiprobably, and a targeted lightning-up with the fire starting from a node with the highest degree. The aim was to find out which one of the two configuration graphs (power-law or Poisson) and under what values of node degree distribution parameters provides maximum node survival in case of a fire. We also introduced the probability of fire transition  $0 which determines if a link becomes inflammable or fire resistant. In [4–6] this probability was predefined and equal for each link. In [4] we estimated how the number of surviving nodes depends on the parameter <math>\tau$  of the node degree distribution (1) and the probability p for both cases of fire ignition. The results allowed to compute the values of the parameter  $\tau$  for which the number of surviving nodes reaches its maximum  $\tau_{max}$ , and to find the following relations between  $\tau_{max}$  and p in cases of random ignition (5) and targeted lightning-up (6):

$$\tau_{max} = 14.2 - 11.8p + 11.6\ln p, \qquad R^2 = 0.98,\tag{5}$$

$$\tau_{max} = 5.7 - 3p + 3.2 \ln p, \qquad R^2 = 0.98. \tag{6}$$

Similar experimental results for configuration graphs with Poisson node degree distribution (2) were conducted in [6]. The resulting relations between  $\lambda_{max}$ and p in cases of random ignition (7) and targeted lightning-up (8) look as follows:

$$\lambda_{max} = 30.6 - 171.1p + 341.7p^2 - 232.8p^3, \qquad R^2 = 0.99, \tag{7}$$

$$\lambda_{max} = 49.6 - 324.7p + 735.4p^2 - 560.6p^3, \qquad R^2 = 0.99. \tag{8}$$

The way to follow was to consider a fire propagation process on the same configuration graphs in so called random environment, meaning that the probabilities of fire transition are not equal for all graph links but follow the standard uniform distribution. The optimal value of the parameter  $\tau$  that ensures maximum node survival in case of a fire for power-law graphs with a random fire ignition was estimated as 1.99 and for targeted lightning-up as 1.12. Simulation results for Poisson configuration graphs yielded the optimal values of  $\lambda$  equal to 1.01 and 1.34 for random and targeted fire start, respectively.

We also considered an issue of comparing the two configuration graph models – power-law and Poisson – in order to determine which one would show a "better" node survival in case of a fire. The results of the following analysis with predefined probabilities of fire transition p (see [5, 6]) showed that in case of a random fire ignition the power-law node degree distribution (1) of a configuration graph ensures a better node survival compared to Poisson graphs. However, in the case of a targeted lightning-up the result depends on the following condition:

$$p(N) = \frac{881.8}{2041.7 - N}, \qquad R^2 = 0.99, \tag{9}$$

which means that if p > p(N) the power-law configuration graphs will be more robust to the fire, otherwise – graphs with Poisson node degree distribution will ensure a better survival of nodes.

A similar comparative study of the same graphs in a random environment showed that in both cases of fire ignition power-law configuration graphs are more resilient to fire than graphs with Poisson node degree distribution.

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# On the Models of Random Multiple Access with Stochastic Energy Harvesting<sup>\*</sup>

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Abstract. We consider various models for the classical synchronised multiple access system with a single transmission channel and a randomised transmission protocol (ALOHA). We assume in addition that there is an energy harvesting mechanism, and any message transmission requires a unit of energy. We study by means of simulations the (in)stability conditions for such models.

Keywords: random multiple access; stochastic energy harvesting; (in)stability.

Nowadays, idea of usage of energy harvesting in communication systems is of great interest because of various application. For example, sensor networks with rechargeable batteries with harvesting energy from environment can significantly extend lifetime of the system. Another example of using energy harvesting was described in [1], where the authors consider a multi-user system with the base station which employs technology OFDM and a wireless channel to transmit both data and energy to its users. The usage of an energy harvesting mechanism in systems with random multiple access (RMA) presents new challenges and, in particular, in determining their stability regions. In work [2] the authors consider models with decentralised energy harvesting mechanism: individual power supplies for a finite number transmitting nodes and studied their stability properties. In the contrast to this work with separate power supplies per each transmitting node and a finite number of transmitting nodes, we (see [3]) considered a common renewable power supply for all users and infinite number of transmitters and found the stability conditions. We remind that that classical ALOHA (see, e.g., [4]) algorithm with infinite number of users is unstable and stable for finite number of users with some condition on input stream of messages. Thus we proved in [3] that an additional energy limitation may stabilise the system.

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At the same time the model from [3] is very simplistic and quite far from the practice (in comparing with [2]). Meantime, if we consider infinite number of users in the model from [2] the system may lose stability.

In this work we study conditions on energy harvesting algorithm for (in)stability the model with infinite number of users and individual power supplies for each of them.

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# On the Insensitivity of Stationary Characteristics to the Service Time Distribution in Queuing System with Limited Resources \*

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**Abstract.** In modern wireless networks, such as Wi-Fi, Wimax or LTE, each user session occupies some amount of radio resources, which depends on various parameters: required data rate, distance between base station and user equipment, signal-to-noise ratio, etc. Such networks can be modeled as a multi-server queue with losses caused by lack of system resources. Classic Erlang or Kelly models deals with queueing models in which each customer has fixed resource requirements. More accurate results can be obtained using systems with random resource requirements. In this work we assume that arrival process is Poisson and resource requirement of each customer is a random variable with given cumulative distribution function. For such systems, we prove the insensitivity of the stationary characteristics to the service time distributions.

**Keywords:** multi-server queuing system; limited resources; random resource requirement; insensitivity of stationary distribution, LTE.

### 1 Introduction

Growing popularity of multimedia services in modern wireless networks forces to develop effective radio resources allocation techniques and methods for estimation of Quality of Service parameters. Classic Erlang and Kelly models do not take into account significant aspect of wireless networks functioning. Amount of radio resources required for each session, i.e. frequency band or transmit power of frequency amplifier, depends not only of required data rate, but also of the distance between mobile terminal and base station, any obstacles between them, etc.

Considering this aspect, we analyze performance measures of wireless networks using multi-server queuing systems with random resource requirements [2,

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**Fig. 1.** N-server queuing system with losses caused by lack of resources with total amount R.

3]. Random process that describes behavior of the system should keep track of amount of resources occupied by each customer, since the amount of occupied resources is random. These queues were first studied in [5], where stationary characteristics of the system were obtained. In [1] an approach for analysis of systems with general arrival and service processes was proposed: instead of tracing amount of resources occupied by each customer, the simplified model keep track of only total amount of occupied resources. The simplification significantly reduced state space and analysis complexity. In [3] simulations showed that for a variety of arrival processes and service time distributions stationary characteristics of initial and simplified model are very close to each other.

In [2] it was analytically shown that for Poisson arrival process and exponentially distributed service times stationary distributions of number of customers and occupied resources for both systems are exactly the same. In this work, we continue study of simplified model and prove analogue of Sevastyanov's theorem [4] for insensitivity of the stationary characteristics to the service time distributions.

### 2 Models Description

Consider a multi-server queuing system with  $N < \infty$  servers, where customers occupy some amount of resources with total finite quantity R for all the serving time. Assume Poisson arrival process with rate  $\lambda$  and customer service times are independent of each other and independent of arrival process.

Arriving customer occupies random value  $r_i$  resources and total amount of occupied resources is increased by  $r_i$ , if there is not enough resources the customer is lost. Random variables  $r_i$  are mutually independent and have cumulative distribution function (CDF) F(x). At time  $\tau_i$  of the departure of a customer, total amount of occupied resources is decreased by random variable  $\nu_i$ . Having number of customers in the system  $\xi(\tau_i)$  and total amount of occupied resources  $\delta(\tau_i)$ , random variables  $\nu_i$  are independent of previous system behavior and have CDF  $F_k(x|y) = P(\nu_i \leq x|\xi(\tau_i) = k; \delta(\tau_i) = y), x \leq y$ , where  $F_k(x|y) = P(\nu_i \leq x|\xi(\tau_i) = y), x \leq y$ . In case of exponential service time distribution, formulas for stationary probability distribution of the system are as follows [2]:

$$Q_{k}(x) = \lim_{t \to \infty} P\{\xi(t) = k ; \delta(t) \le x\} = q_{0}F^{(k)}(x)\frac{\rho^{k}}{k!}, \ 0 \le x \le R, \ 0 < k \le N,$$
(1)
$$q_{0} = \left(1 + \sum_{k=1}^{N} F^{(k)}(R)\frac{\rho^{k}}{k!}\right)^{-1},$$
(2)

where  $F^{(k)}(x)$  is k-fold convolution CDF F(x). We will show below that formulas (1) and (2) hold true for any service time distribution B(x) with finite mean value.

Behavior of the system can be described by Markov process  $(\xi(t), \delta(t), \beta(t))$ , where  $\beta(t) = (\beta_1(t), \beta_2(t), \ldots, \beta_{\xi(t)}(t))$  is vector of elapsed service times of each customer. Let us denote  $P_t$  - probability distribution at time t and  $P_0$ - initial distribution at t = 0. Assume that distribution  $P_0$  is symmetric on  $(\xi(t) = k, \delta(t) \leq x)$  subspace with regard to variables  $y_1, y_2, \ldots, y_k$ , then  $P_t$  is also symmetric on  $(\xi(t) = k, \delta(t) \leq x)$ .

**Lemma 1.** For any distribution  $P_0$ , distribution  $P_t$  has k-dimensional probability density function (PDF)  $Q_k(x, y_1, ..., y_k; t)$  at  $(\xi(t) = k, \delta(t) < x, y_1, y_2, ..., y_k; t)$ if  $t > \max(y_1, y_2, ..., y_k)$ , and

$$Q_k(x, y_1, y_2, \dots y_k; t) \le \lambda^k F^{(k)}(x) \prod_{i=1}^k [1 - B(y_i)], \quad 1 \le k \le N.$$
(3)

*Proof.* Following inequalities hold true

$$P \{\xi(t) = k, \delta(t) < x, y_i < \beta_i(t) < y_i + \Delta_i, \quad 1 \le i \le k\} = P(A) \le \le F^{(k)}(x) \prod_{i=1}^k [1 - B(y_i)] [1 - e^{-\lambda \Delta_i}] \le \lambda^k F^{(k)}(x) \prod_{i=1}^k [1 - B(y_i)] \Delta_i,$$

since  $1-e^{-\lambda\Delta_i} \leq \lambda\Delta_i$ , and for occurance of the event A customers have to arrive at time intervals  $(t - (y_i + \Delta_i), t - y_i)y_i, i = 1, 2, ..., k$ , with service times at least  $y_i, i = 1, 2, ..., k$ , and total amount of occupied resources by arrived customers doesn't exceed x. Thus, PDF existence and inequality (3) are proved.

Transition probabilities in time interval  $\Delta t$  have the following form:

$$Q_0(0;t + \Delta t) = Q_0(0;t) (1 - \lambda F(R)\Delta t) + + \int_0^R \int_0^\infty Q_1(dx, y_1;t) \frac{B(y_1 + \Delta t) - B(y_1)}{1 - B(y_1)} dy_1 + o(\Delta t);$$
(4)

$$Q_{k}(x, y_{1}, \dots, y_{k}; t + \Delta t) = \int_{0 \leq y \leq x} Q_{k}(dy, y_{1} - \Delta t, \dots, y_{k} - \Delta t; t) \times \\ \times (1 - \lambda F(R - y)\Delta t) \prod_{1 \leq j \leq k} \frac{1 - B(y_{j})}{1 - B(y_{j} - \Delta t)} + (k + 1) \int_{x \leq y \leq R} (1 - F_{k}(y - x|y)) \times \\ \times \int_{0}^{\infty} \left[ Q_{k+1}(dy, y_{1} - \Delta t, \dots, y_{k} - \Delta t, y_{k+1}; t) \frac{B(y_{k+1} + \Delta t) - B(y_{k+1})}{1 - B(y_{k+1})} \right] dy_{k+1} \times \\ \times \prod_{1 \leq j \leq k} \frac{1 - B(y_{j})}{1 - B(y_{j} - \Delta t)};$$
(5)

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$$Q_N(x, y_1, \dots, y_k; t + \Delta t) = Q_N(x, y_1 - \Delta t, \dots, y_k - \Delta t; t) \prod_{1 \le j \le N} \frac{1 - B(y_j)}{1 - B(y_j - \Delta t)}.$$
(6)

Let us denote  $Q_k^*(x, y_1, y_2, \dots, y_k; t) = \frac{Q_k(x, y_1, y_2, \dots, y_k; t)}{[1-B(y_1)][1-B(y_2)]\dots[1-B(y_k)]}$ . Assume existence of partial derivatives  $\frac{\partial Q_k^*}{\partial t}$ ,  $\frac{\partial Q_k^*}{\partial y_i}$ ,  $1 \le i \le k$ ,  $0 \le k \le N$ , then using (4) – (6) we obtain following differential equations:

$$\frac{\partial Q_0^*(0)}{\partial t} + \lambda Q_0^*(0) F(R) = \int_{0 \le x \le R} \int_0^\infty Q_1^*(dx, y_1; t) dB(y_1), \tag{7}$$

$$\frac{\partial Q_k^*(x)}{\partial t} + \frac{\partial Q_k^*(x)}{\partial y_1} + \dots + \frac{\partial Q_k^*(x)}{\partial y_k} + \lambda \int_{0 \le y \le x} Q_k^*(dy, y_1, \dots, y_k; t) F(R - y) = \\ = (k+1) \int_{x \le y \le R} \left(1 - F_k(y - x|y)\right) \int_0^\infty Q_{k+1}^*(dy, y_1, \dots, y_{k+1}; t) dB(y_{k+1}),$$

$$(8)$$

$$\frac{\partial Q_N^*(x)}{\partial y_k} + \frac{\partial Q_N^*(x)}{\partial y_k} + \frac{\partial Q_N^*(x)}{\partial y_k} = 0$$

$$\frac{\partial Q_N^n(x)}{\partial t} + \frac{\partial Q_N^n(x)}{\partial y_1} + \ldots + \frac{\partial Q_N^n(x)}{\partial y_k} = 0, \tag{9}$$

with boundary condition

$$\lambda \int_{\substack{0 \le y \le x \\ 0 \le k \le N-1.}} F(x-y) Q_k^*(dy, y_1, y_2, \dots, y_k; t) = (k+1) Q_{k+1}^*(x, y_1, \dots, y_k, 0; t),$$
(10)

We can make sure by substitution that stationary solution of system of equations (7) - (9) with boundary condition (10) is

$$Q_k^*(x, y_1, y_2, \dots, y_k) = Q_0^*(0) \frac{\lambda^k}{k!} F^{(k)}(x),$$
(11)

$$Q_0^*(0) = \left(1 + \sum_{k=1}^N \frac{\lambda^k}{k!} F^{(k)}(R)\right)^{-1}.$$
 (12)

Thus, we proved the following theorem.

**Theorem 1.** If service time distribution with CDF B(x) have finite mean value b > 0, then stationary probability distribution of random process  $(\xi(t), \delta(t), \beta(t))$  is

$$Q_k(x, y_1, y_2, \dots, y_k) = \lim_{t \to \infty} P\{\xi(t) = k ; \delta(t) \le x; \beta_1(t) < y_1, \dots, \beta_k(t) < y_k\} = q_0 F^{(k)}(x) \frac{\rho^k}{k!} [1 - B(y_1)] \dots [1 - B(y_k)], \quad 0 \le x \le R, \ 0 < k \le N,$$
(13)

where  $\rho = \lambda b$  and  $q_0$  is given by formula (2).

In particular, it follows that stationary probability distribution of random process  $(\xi(t), \delta(t))$  is also determined by formulas (1) and (2) as in case of exponential service time distribution. Hence, it is insensitive to service time CDF.

#### 3 Conclusion

We considered multi-server queuing system with losses caused by lack of system resources. It was proved that stationary probability distribution of the system in case of Poisson arrival process is insensitive to service time distribution. The insensitivity property allows to develop simple rules for wireless networks performance evaluation that do not require detailed knowledge of traffic statistics.

In our further research, we will check if this result holds true for MMPP arrival process.

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# A Simulation Based Analysis of SINR for Device-to-Device Communications in Circular Clusters \*

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**Abstract.** In this contribution we address a common scenario for deviceto-device communications. Using the simulation approach we analyze the interference from a single interferer. The proposed model serves as a basis for general analysis featuring more than a single interferer.

Keywords: SIR, SINR, wireless network, LTE, D2D

### 1 Introduction

Modern technologies in wireless networks (cellular, WLAN) allow devices to communicate directly without using a base station or wireless access point. This type of communication called device-to-device (D2D). D2D communications may increase the spatial reuse of radio resources and therefore may increase the total throughput of wireless systems. A scheduler responsible for radio resource allocation to communicating devices is expected to be an integral part of future wireless systems as two communicating devices using the same radio resource may cause interference and subsequent rate reduction. The scheduler should allocate resources in such a way that stations communicating directly minimally interfere with each other while radio resources are efficiently utilized.

An ultimate metric for assessing the signal quality in wireless systems is signal-to-interference-plus-noise ratio (SINR). In those cases when noise is constant signal-to-interference ratio (SIR) is commonly used. In our research we investigate different D2D interaction schemes and estimate the value of SIR for each of those. In this short communication we will focus on one particular example.

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Fig. 1. Considered scheme with circular clusters

#### 2 Circular Clustering in D2D

Consider a D2D scheme with seven circular clusters as shown in Fig. 1. Every cluster contains a communicating devices pair (transmitter and receiver). The transmitter located in the center of the cluster and the receiver is distributed randomly and uniformly within the cluster. Cluster radius determines the maximum communication distance between receiver and transmitter. In our case we assume that radii of all considered clusters are equal.

We assume that all of the considered transmitters use the same radio resource with equal output power. Therefore interference occurs between transmitted signals. The clusters in our scheme are not overlapping because we need the receivers to be closer to their own transmitters than to the neighboring ones. The clusters are contiguous resulting in the worst interference as transmitters in the neighboring clusters are located as close as possible.

Let us denote the central cluster as the main and other six clusters as the interfering. Then the receiver of the main cluster,  $Rx_0$ , in addition to the incoming signal of interest from  $Tx_0$  receives signals from the transmitters of the interfering clusters  $(Tx_1, \ldots, Tx_6)$ . We evaluate SIR for  $Rx_0$  according to

$$SIR = \frac{S}{\sum_{i=1}^{6} I_i} \tag{1}$$

where S is the power of the signal of interest and  $I_i$  is the power of the interfering signal.

The propagation models in the nominator and denominator of (1) are assumed to be



Fig. 2. Distribution function of SIR obtained using simulation

$$S = S(l_0) = g_0 l_0^{-\gamma_0}, I_i := I_i(l_i) = g_i l_i^{-\gamma_i}$$
(2)

where  $g_i$  is the power of  $Tx_i$   $(i = \overline{0,6})$ ,  $l_i$  is the distance between  $Tx_i$   $(i = \overline{0,6})$  and  $Rx_0$ ,  $\gamma_i$  is the pass loss exponent [1,2]. Substituting (2) into (1) and simplifying noticing that  $g_i$  for all i are equal we get the following expression for SIR

$$SIR = \frac{l_0^{-\gamma_0}}{\sum_{i=1}^6 l_i^{-\gamma_i}}$$
(3)

#### 3 Numerical Analysis

Assuming that the receiver  $Rx_0$  is uniformly distributed within the main cluster we developed a simulator and determined the distribution of SIR according to (3). The cumulative distribution function (CDF) is illustrated in Fig. 2. For a given value  $SIR^*$  it gives the probability that  $SIR < SIR^*$  at  $Rx_0$ . It is interesting to note that CDF does not depend on the radii of clusters if they are equal to each other. Further, CDF is also insensitive to the emitter power of transmitters. Using the CDF one could evaluate the mean value, variance and quantiles for  $\alpha = 0.05; 0.1; 0.15; 0.2$  of SIR.

#### 4 Conclusion

In this paper we reported on the simulation model capable of evaluating SIR distribution for D2D communications. One could use the proposed model for assessing the link quality metric in terms of mean SIR values, variance of SIR and outage probabilities corresponding to lower quantiles of SIR distribution. In our future study we aim to develop an analytical model for considered scenario

obtaining closed-form approximation for moments and SIR distributions. Due to dependence between distances involved in the nominator and denominator of (1) various approximations will be considered. The first approach would be to consider all the distances from interference to the receiver of interest following the same distribution. According to the second approach, the interference created by nodes communicating over the same channel can be approximated by Normal distribution. The ratio between the useful signal and the sum of interfering ones can then be obtained using classic approach for finding functions of random variables, see e.g. [1, 2].

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# On Numerical Estimation of SINR for Square Wireless Clusters\*

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**Abstract.** The signal-to-interference-plus-noise (SINR) ratio experienced by a mobile station is the ultimate metric determining the throughput that it receives using a certain wireless technology of interest. In this work, we propose an analytical model for SINR estimation in a rectangularly shaped clusters for both uplink and downlink cases, showing that in both these cases the closed-form expressions can be derived. We numerically elaborate on the results highlighting important dependencies and trade-offs for the chosen typical scenario.

### 1 Introduction and System Model

The signal-to-interference-plus-noise (SINR) ratio is one of the most important metrics pertaining to the channel quality and describing the performance of wireless systems, i.e. the maximum throughput that a user obtains using the wireless channel. The SINR characterizes a wireless channel between the transmitting and the receiving devices, i.e. it is a ratio of the useful energy to the interference-and-noise. Due to the users' mobility, the SINR in wireless systems is often a function of the current position of the user and thus can be considered as a random variable [1].

Our proposed framework allows for obtaining the closed-form expressions for the SINR values. We assume that the mobile devices are located in the environment composed of adjacent rectangular clusters of certain sides. The wireless access points are located in the centers of the clusters. Mobile devices are uniformly distributed over the corresponding areas. Although we do not focus on a particular radio technology, we take a set of assumptions on its specific details. First, mobile nodes and access points operating in adjacent clusters are using the same set of frequencies to communicate and interfere (in Wi-Fi – the same

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Fig. 1. An illustration of the model for SINR analysis.

channel; in micro-LTE – the same resource blocks). Our model thus corresponds to the worst-case interference scenario.

Formally, SINR can be expressed as:  $SINR = \frac{S}{\sum_{i=1}^{N} I_i + \sigma^2} = \frac{gl^{-\alpha}}{\sum_{i=1}^{N} gl_i^{-\alpha_i} + \sigma^2}$ , where the received signal power S is a function of the distance between the transmitter and the receiver and the interference power  $I_i$  is a function of the distance and the signal between the receivers and the  $i^{th}$  interfering device. Further, N is the number of interfering sources,  $\sigma^2$  is the noise power, g is the transmission power assumed to be constant for all the transmitters, l is the distance of interest, and  $\alpha$  is the path loss exponent that ranges from 2 to 6 [2].

In what follows, we assume the noise to be zero and investigate the signalto-interference (SIR) ratio. We note that the received signal is mainly affected by the signals transmitted at the same and the neighboring frequencies. To this end, Fig. 1 shows the considered scenario, where the tagged cluster contains the *target* receiver, while the interfering node is located in an adjacent cluster called the *interfering cluster*. Let the clusters be of square shape with the side length c. We define a pair of communicating devices  $(Tx_0, Rx_0)$  as the *tagged* one. And the distance between them is denoted as  $R_0$ , the distance between the interfering devices  $Tx_1$  and  $Rx_0$  is  $D_1$ .

 $SIR = \frac{S(R_0)}{I(D_1)} = \frac{gR_0^{-\alpha_1}}{gD_1^{-\alpha_2}} = \frac{R_0^{-\alpha_1}}{D_1^{-\alpha_2}}$ . Using the method described in [3], we establish the SIR PDF expression in the integral form (1). Due to the page limitation, we do not include the final expression here:  $W(y_1) := \int_0^\infty \sum_{i=1}^3 I_i(y_1, y_2) dy_2$ .

### 2 Numerical Results and Conclusions

We validate our analysis with simulation data and conclude that there is a perfect match. This allows to further rely on the analytical model. The considered scenario has a number of properties that could be demonstrated by using the proposed model. In particular, it is insensitive to the choice of some input parameters. Fig. 2(a) shows the effect of different values of  $\alpha$  for the downlink scenario with the cluster dimensions set as a = b = 1. The same  $\alpha$  is again used for both interferer-to-receiver and transmitter-to-receiver paths. The value of  $\alpha$ provides the scaling effect on the resulting SIR density. Fig. 2(b) highlights the



(c) UL: The effect of  $\alpha$  (a = b = 1) (d) UL: Dimension effect ( $\alpha_1 = 2$ )

Fig. 2. SINR PDF as a function of system variables

insensitivity of the model to the dimensions of interest. The results are demonstrated for a = b = c, where  $c \in [1, 2, 3]$ ,  $\alpha = 2$ . These properties hold for the downlink scenario as well, i.e. for different  $\alpha$  and the sets of a and b.

Fig. 2(c) shows the effect of different values of  $\alpha$  in the uplink scenario with  $\alpha_1$  corresponding to the propagation path between the transmitter and the receiver,  $\alpha_2$  – to the one path between the interferer and the receiver. This effect is fairly straightforward and self-explanatory: the larger the  $\alpha_2$  is, the better the interference picture at the receiver becomes. The value of  $\alpha_2$  is mainly dictated by the wall material, i.e. we can compute the SIR for different materials of walls. Our numerical investigation reveals that for square configurations the SINR value is insensitive to the side length of a square, see Fig. 2(d). Further, the effect of the wall material affecting the propagation exponent could be dramatic. In summary, we conclude that we can estimate the SINR distribution analytically without the need for time-consuming simulations.

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# Modeling Joint Uplink Scheduling of M2M and H2H Transmissions over 3GPP LTE Cellular Networks<sup>\*</sup>

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Abstract. In this work, we study a cellular system (e.g., 3GPP LTE-Advanced), which concurrently serves machine-to-machine (M2M) and human-to-human (H2H) traffic. Our study focuses on uplink transmissions of M2M and H2H devices sending their data "files" and streaming sessions to the network, respectively. Together with traffic dynamics, we account for the spatial distribution of M2M and H2H devices on the plane, which allows us to characterize their expected performance analytically across a number of metrics. Our system-level simulations confirm the validity of the proposed system model.

### 1 Introduction

The emergence of novel technological paradigms together with ubiquitous connectivity will bring us to a networked society in foreseeable future. Targeting massive connectivity of identifiable objects, the world of heterogeneous unattended devices referred to as the *Internet of Things (IoT)* has been attractive for both industry and academia, which are investing large amounts of resources into developing it. In turn, *machine-to-machine (M2M)* communication is predicted to account for the largest submarket within the IoT market by 2020 [1], [2]. Along with legacy cellular technologies, the recent 3GPP Long Term Evolution-Advanced (LTE-Advanced) system has been instrumental to enable M2M performance and applications [3]. However, the available capacity of LTE signaling

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channel may deteriorate for massive M2M connectivity on the one hand and negatively affect the expected QoS requirements for the conventional humanto-human (H2H) devices on the other [4]. While developing an M2M-enabled LTE scheduling mechanism that handles the overloaded traffic efficiently, there have been many partial solutions proposed [5], [6], [7]. Taking into account the typical M2M traffic properties, these research works have concentrated mostly on simulation-level design, as well as fall short of incorporating the geometric spatial distribution when it comes to characterizing the system behavior. As an extension to our previous work [8], we specifically study the impact of M2M traffic on the system capacity accounting for H2H communication and respective QoS requirements with a joint M2M/H2H scheduling scheme. The provided approximation for the main performance metrics has been verified by extensive simulations. The remainder of this text is as follows. In Section 2, we describe the system model. Section 3 details the main analytical approach and the metrics of interest. Finally, we present selected numerical results and conclusions in Section 4.

### 2 System Model

This section describes the important system entities under consideration and introduces our main assumptions associated with them.

**General Description.** Let us consider a centralized wireless architecture (illustrated in Fig. 1) consisting of (i) a single base station (BS) located at the center of a circular cell and (ii) a number of identical M2M devices and H2H devices spatially distributed within the cell and associated with the BS.

Together with the spatial component, we introduce traffic dynamics, when the connection between the M2M/H2H device and the BS (termed *session*) exists for a certain random time interval and also depends on the connection type. In particular, M2M devices associated with the BS transmit data "files", whereas H2H devices require voice or web streaming sessions with predefined rate requirements. The data transmissions of both types share the same LTE uplink channel, and scheduling requests are managed centrally at the BS. The scheduling procedure as part of Radio Resource Management (RRM) mechanism is performed based on the connection type and the available channel resources fulfilling the QoS requirements or declining the session. In what follows, we detail



Fig. 1. Topology of the envisioned system: connected M2M and H2H devices.

the key assumptions prior to explaining the mathematical derivations behind the proposed model.

**Topology Distribution.** Both M2M and H2H devices are assumed to be spatially distributed within the cell of radius R, so that the probability density function (PDF) for the distance to the BS is given by  $f_d(d)$ . Moreover, we consider a special case when the devices are distributed according to a Poisson Point Process (PPP) on the plane and thus are uniformly deployed within the area of interest. In that case, the distribution of distance within the circle of radius R is defined by the expression  $f_d(d) = \frac{2d}{R^2}$ .

**Power-Rate Mapping and Signal Propagation.** We assume that the instantaneous data rate for the M2M/H2H device obeys the Shannon's formula as  $r(d) = w \log (1 + \gamma(d) p)$ , where w denotes the spectral bandwidth, p is the transmit power, and  $\gamma(d)$  is a function of distance d reflecting the signal-noise ratio per a unit of power. The interference from the neighboring cells is assumed to not exceed the noise level. Avoiding infinite data rates, we refer to the maximum data rate for a session as to  $r_{\text{max}}$ . To characterize the signal propagation over the wireless medium, we employ the following power model:

$$\gamma\left(d\right) = \min\left[\frac{G}{d^{k}N}, \gamma_{\max}\right],\tag{1}$$

where k is the propagation exponent, G is the propagation constant, N is the noise power, and  $\gamma_{\max}$  represents the maximum path gain taken with respect to the maximum data rate  $\gamma_{\max} = \frac{1}{p} \left( e^{\frac{r_{\max}}{w}} - 1 \right)$ . Equation for  $\gamma (d)$  implies that if the distance to the BS becomes less than a certain threshold  $d_{\min} = \left(\frac{G}{\gamma_{\max}N}\right)^{\frac{1}{k}}$ , the instantaneous data rate as well as the path gain remain constant. Thereby, knowing the maximum data rate  $r_{\max}$ , the maximum path gain  $\gamma_{\max}$ , and the distance threshold  $d_{\min}$  by the above expressions, we may define the path gain  $\gamma$  and the instantaneous data rate r as follows:

(i) 
$$r(d) = r_{\max}$$
,  $\gamma(d) = \gamma_{\max}$ ,  $0 \le d \le d_{\min}$ ;  
(ii)  $r(d) = w \log\left(1 + \frac{pG}{d^k N}\right)$ ,  $\gamma(d) = \frac{G}{d^k N}$ ,  $d_{\min} < d \le R$ . (2)

Arrivals and QoS Requirements. Both M2M and H2H arrivals into the system follow a Poisson process of intensity  $\lambda_m$  and  $\lambda_h$ , correspondingly. A new M2M device targets to transmit a single data file of size s, which is distributed exponentially with the average  $\theta$ . Importantly, any M2M data transmission requires the minimal bitrate  $b_m$ . However, the actual bitrate may exceed this threshold. If admitted, the M2M device is served by the BS until its entire file is transmitted. In contrast, each H2H device requires a fixed bitrate  $b_h$  during the time interval of exponential duration (with the average  $\mu^{-1}$ ). Once the H2H device arrives into the system, the latter allocates the needed resource, and the former is served continuously until its sension ends.

Scheduler and Power Control Properties. Naturally, the BS is assumed to schedule the requests sent by both M2M and H2H devices within a specified resource pool, which we assume being 1 without the loss of generality. As referred above, the individual shares of the resource have to be assigned such that they satisfy both M2M and H2H QoS requirements. The actual data rate may be calculated as a product of the random instantaneous rate  $r_m(d)$  (M2M),  $r_h(d)$  (H2H), and a dedicated share  $\varepsilon$  of the total resource. Both instantaneous rates  $r_m(d)$ ,  $r_h(d)$  are obtained based on the respective power policy, which in our study (i) for M2M translates into the fixed power  $p_m$  for simplicity and (ii) for H2H leads to SNR threshold power control, when the power is either set to achieve SNR  $\eta$  (and the respective rate  $r_{\eta}$ ) within a circle of radius  $d_{\eta}$  or to the maximum level  $p_{\text{max}}$ . Therefore, the system settings may be summarized as follows:

(i) 
$$p_m(d) = p_m = const$$
,  $r_m(d) = \min[w \log(1 + \gamma(d) p_m), r_{\max}]$ ,  
(ii)  $p_h(d) = \min\left[\frac{\eta}{\gamma(d)}, p_{\max}\right]$ ,  $r_h(d) = \min[w \log(1 + p_{\max}\gamma(d)), r_{\eta}]$ .
(3)

1) M2M request scheduling. For all the data sessions generated by M2M devices, the actual time spent in the system until service completion depends on the variable actual data rate. Naturally, the actual data rate for one device is a function of allocated resource, which is assigned to a set of devices as an integer number of identical chunks of size c < 1. All  $n_m$  devices in service share the allocated resource so that the individual share for one device equals  $\frac{1}{n_m} z(n_m) c$ , where  $z(n_m)$  is the minimum possible number of chunks to guarantee  $b_m$  for everyone. Clearly, if  $B(n_h)$  is the share of total resource occupied by all ongoing H2H sessions, then:

$$z\left(n_{m}\right)c+B\left(n_{h}\right)\leq1.\tag{4}$$

Moreover, each new M2M device requires the rate of at least  $b_m$ , or, equivalently, the share  $\frac{b_m}{r_m}$  of the total resource. Therefore, the condition  $z(n_m) \leq \frac{b_m n_m}{r_{\max}c}$ should be satisfied for any device out of  $n_m \leq N_m^{\max}$  (the maximum number of M2M devices). Otherwise, the new device cannot be served and is blocked by the system.

2) H2H request scheduling. H2H divices are guaranteed to obtain the exact rate  $b_h$ , so as to maintain their target QoS. Additionally, all H2H sessions in service together with M2M traffic utilize not more than the total available resource 1, i.e. (4) is satisfied. Correspondingly, the BS is able to accept a new H2H request if it does not violate the minimum data rate requirement on the one hand, and if there are still resources available to the new requests as well as their number does not exceed the maximum number of H2H devices in service  $N_h^{\text{max}}$ on the other hand. The new request is blocked and considered lost permanently if any of the above conditions is not met.

#### **3** Performance Analysis

In this section, we provide our analysis allowing to evaluate the main performance metrics, such as the average number of M2M and H2H devices as well as the corresponding blocking probabilities.

General Remarks. We remind that according to our assumptions the interarrival times for both M2M and H2H traffic as well as the M2M file size and the duration of the H2H session are exponentially distributed. Exploiting this fact, we may describe the system behavior by means of a Markov process  $\boldsymbol{X}(t)$ , the state of which is defined by all currently running M2M and H2H sessions. Specifically, as the key parameter we select the individual distance between the transmitter (either M2M or H2H device) and the BS. We note that a set of such distances for all ongoing sessions determines the behavior of the entire system. Consequently, we describe the state of the process  $\boldsymbol{X}(t)$  as  $\boldsymbol{x} = ((d_1^h, \ldots, d_{n_h}^h), (d_1^m, \ldots, d_{n_m}^m))$ , where  $n_m$  and  $n_h$  are the current numbers of M2M and H2H devices, correspondingly, while  $d_i^m$   $(d_i^h)$  are the distances between the M2M (H2H) device i and the BS.

Aggregated Markov Process. We note that the process X(t) features the uncountable number of states, which makes it rather complex to analyze straightforwardly. In order to tackle the problem at hand and decrease the number of states we employ the state aggregation technique. Aggregating the states of the initial process X(t) by the number of M2M and H2H, we eventually arrive at the aggregated Markov process Y(t), where the state is defined as  $y = (n_h, n_m)$ . Fig. 2 illustrates the general structure of the aforementioned process and, in particular, the selected state  $(n_h, n_m)$  together with the corresponding transitions  $Q[(n_h, n_m), (\bullet, \bullet)]$ .



**Fig. 2.** State transition diagram for the aggregated Markov process  $\boldsymbol{Y}(t)$ : a particular state  $(n_h, n_m)$  and structure of the state space  $\mathcal{Y}$ .

Hence, we construct the process such that the state transitions would reflect the spatial nature of the system under consideration and provide a spatially averaged estimate of the system performance. In order to find the steady-state distribution of the process  $\mathbf{Y}(t)$ , we derive the infinitesimal generator matrix  $\mathbf{Q}$ . While the backward transition from the state  $(n_h, n_m)$  to the state  $(n_h - 1, n_m)$ is near-trivial and equals  $Q[(n_h, n_m), (n_h - 1, n_m)] = i\mu$ , the rest of the transitions are more elaborate and deserve separate attention.

**Performance Measures of Interest.** Having obtained the transition intensities  $Q[(\bullet, \bullet), (\bullet, \bullet)]$ , we compose the infinitesimal matrix Q. Solving the corresponding system of equilibrium equations, we may finally establish the steady-state probability distribution  $\{P(n_h, n_m), (n_h, n_m) \in \mathcal{Y}\}$ . Based on it, we may derive any stationary metrics of interest, such as e.g., the average number of

M2M and H2H devices:  

$$E[n_m] = \sum_{n_m=1}^{N_m^{\max}(n_h)} \sum_{n_h=0}^{N_h^{\max}(n_m)} n_m P(n_h, n_m), E[n_h] = \sum_{n_m=0}^{N_m^{\max}(n_h)} \sum_{n_h=1}^{N_m^{\max}(n_h)} n_h P(n_h, n_m).$$

#### 4 Numerical Results and Conclusion

In order to illustrate our analysis with some numerical results, we compare the proposed approximation with the corresponding system-level simulations focusing on the average number of H2H and M2M devices in the LTE-based system under characteristic parameters: R = 100m, w = 20MHz,  $\eta = 23$ dB,  $r_{\rm max} = 5$ Mbps, c = 0.1, and  $\theta = 1.5$  Mbit. To this end, Fig. 3 indicates that both analysis and simulations agree satisfactorily. These numerical results confirm our practical intuition on that the system performance degrades when the offered M2M loading grows resulting in more M2M devices at service, as well as having a lower number of H2H devices capable of transmitting uplink data over the channel.



Fig. 3. Average number of M2M and H2H devices: analysis and simulation

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# Modeling A Load Balancing Scheme between Primary Licensed and LSA Frequency Bands in 3GPP LTE Networks<sup>\*</sup>

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**Abstract.** Mobile data traffic is predicted to have an exponential growth during at least 5 upcoming years. Under these circumstances, new spectrum reuse techniques are being developed. One of these techniques is Licensed Shared Access (LSA) enabling the usage of the same spectrum by two parties. At any moment, the spectrum is used only by one party, so that the access is not simultaneous, but sequential to protect the corresponding services from interference. The party owning the spectrum has priority in spectrum usage at all times. In this work, we consider such a system as a queuing model with reliable and unreliable server pools from the second party's point of view to estimate the impact of the unreliable LSA bands on the resulting system performance. Numerical solutions for the non-linear optimization problem of resource reservation for interrupted users will be covered in the presentation.

#### 1 Introduction

Today, manufacturers of telecommunication equipment, such as Cisco Systems, predict that we will experience near-exponential growth of mobile traffic within the following few years. Forecasts maintain that monthly totals will rise up to 24.3 exabytes within 5 years from now. Under such circumstances, numerous techniques are being developed to mitigate this problem. One of the techniques features Device-to-Device communication (D2D for short) [1]. Another option is to allocate more spectrum for mobile wireless systems. Reallocating frequencies is hard and may disrupt some services that were built around the legacy allocation mechanisms. The use of higher frequencies is also limited, since in that case the channel losses will also become higher, the energy consumption will increase, and we will need to transmit at higher powers.

An alternative option that is currently being developed is named the Licensed Shared Access (LSA). This technique allows us to use more bands without breaking services that conventionally needed these bands for their operation. The idea

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behind LSA is such that frequency bands are rented from their original owner and are then used by e.g., a cellular operator. A dedicated regulator entity keeps hold of the database with the information on the rented bands, as well as their status, and also acts as an interface between the operator and the original spectrum owner [2]. If the original owner needs to use these bands, it notifies the regulator entity of the respective needs, and the operator must in turn vacate the rented bands and keep its users from utilizing them. In that case, some users may be relocated to the bands that operator owns reliably, thus redistributing load in the network. After the owner finishes using the bands, it notifies the regulator entity that the bands are now vacant again. After that, the operator may continue to use its rented frequencies [3]. This LSA system has a downside of being unreliable – at any given moment the bands can be revoked by the owner and the operator will have to redistribute the available resources for its users.

### 2 Queuing Model with Unreliable Servers

Let us consider a single LTE-network cell with LSA as a queuing system with two server pools. The primary and the LSA bands correspond to the first and the second server pools [4]. If an LSA band revocation (shutdown) occurs when the primary operator band is fully occupied, then the service of the requests in the second pool interrupts and these requests are lost. Thereby, an LTE cell with primary and LSA bands could be described as a multi-server queuing model with unreliable server pool.

We assume that a Poisson flow of rate  $\lambda$  arrives into the system consisting of the two server pools of size  $C_1$  and  $C_2$  correspondingly. Service requests are then received by the servers of the first pool. If all servers  $C_1$  of the first pool are occupied, the requests are forwarded to the second pool. We further assume that the second pool servers become unavailable simultaneously with the rate  $\alpha$ and recover with the rate  $\beta$ . Recovery and failure times follow the exponential distribution. If there are free servers in the system when a request arrives, it occupies one server for a random time specified by the exponential distribution with the parameter  $\mu$ . If there are no free servers, the request is lost. In case of the second pool unavailability, all requests that have been serviced by the second pool are evacuated to the first pool. If some requests cannot be sent to the first pool, they are lost.

Let  $n_1$  be the number of occupied servers in the first pool,  $n_2$  is the number of occupied servers in the second pool, and s is the state of the second pool (s = 1 if the pool is operational and s = 0 if the pool is unavailable). Hence, we can describe the system in question by a Markov process  $\boldsymbol{X}(t) = (N_1(t), N_2(t), S(t))$  with the state space:

$$\mathcal{X} = \{n_1 = 0, \dots, C_1, n_2 = 0, \dots, C_2, s = 1; n_1 = 0, \dots, C_1, n_2 = 0, s = 0\}.$$

We denote the stationary probability distribution as

 $\boldsymbol{p} = (p(n_1, n_2, s) : (n_1, n_2, s) \in \mathcal{X})$ . The key performance measures of our LSA model – the probability  $I_1$  that at least one service will be interrupted and lost

when the second pool becomes unavailable, the probability  $I_2$  that a targeted service will be interrupted and lost when the second pool becomes unavailable, the probability B that an arrived request will be blocked due to all servers being occupied – are calculated as follows:

$$B = p(C_1, C_2, 1) + p(C_1, 0, 0),$$
  

$$I_1 = \sum_{n_2=1}^{C_2} \sum_{n_1=C_1-n_2+1}^{C_1} p(n_1, n_2, 1),$$
  

$$I_2 = \sum_{n_2=1}^{C_2} \sum_{n_1=C_1-n_2+1}^{C_1} \frac{n_2 - C_1 + n_1}{n_2} p(n_1, n_2, 1).$$

To derive the above performance measures, we need to find the stationary probability distribution p. It could be numerically computed as a solution to the system of equilibrium equations  $p \cdot A = 0$ ,  $p \cdot \mathbf{1}^T = 1$ , where  $\mathbf{A}$  is the infinitesimal operator. The expression to produce the elements of this operator is as follows:

$$a((n_1, n_2, s), (n'_1, n'_2, s')) = \begin{cases} \alpha, & \text{if } n'_1 = \min\left(C_1, n_1 + n_2\right), n'_2 = 0, s = s' - 1\\ \beta, & \text{if } n'_1 = n_1, n'_2 = n_2 = 0, s' = s + 1, \\ \lambda, & \text{if } n'_1 = n_1 + 1, n'_2 = n_2, s' = s\\ & \text{or } n'_1 = n_1 = C_1, n'_2 = n_2 + 1, s' = s = 1, \\ n_1 \mu, & \text{if } n'_1 = n_1 - 1, n'_2 = n_2, s' = s, \\ n_2 \mu, & \text{if } n'_1 = n_1, n'_2 = n_2 - 1, s' = s, \\ *, & \text{if } n'_1 = n_1, n'_2 = n_2, s' = s, \\ 0, & \text{otherwise}, \end{cases}$$

where  $* = -(s\alpha + (1-s)\beta + \lambda \cdot 1\{n_1 + sn_2 < C_1 + sC_2\} + (n_1 + n_2)\mu).$ 

#### 3 Impact of LSA Unavailability on Service Interruption

Let us further review the system under investigation. The LTE base station typically has the 10 MHz of primary band that is always available, and could also have the 5 MHz band that is rented under the LSA license. Assuming that we have mobile LTE users that are trying to upload their video files, we may consider that each user requires a data transmission rate of 250 KBytes per second. Hence, the system throughput is 40 MBits per second and 20 MBits per second for the first and the second pools, respectively (the spectral efficiency of LTE is 4 bits/Hz here), and so the capacity of the band is  $C_1 = 20$  users for the first pool and  $C_2 = 10$  for the second pool. The average service time of one user is 15 seconds, the time between the data arrivals is variable. The second pool is revoked every two minutes on average, and its recovery then takes around one minute. To this end, Fig. 1 outlines the main characteristics (performance measures) of the considered LSA system. The figure indicates the load of above 0.7, as for lower loads the interruption probability reaches the values of under  $10^{-8}$ .



Fig. 1. Considered interruption probabilities

#### 4 Conclusion and Further Studies

The proposed model allows us assessing the reliability of the considered LSA system in terms of interruption probability under different loads. In our further studies, we plan to analyze other performance measures, such as the system utilization coefficient, the mean number of users in the system, and the probability that a service would not be interrupted if the second pool becomes unavailable. In addition, we plan to build a simulation model of the described system and compare the results derived from our queuing model against the respective simulation results. Also, we can consider a situation when the operator does not have to interrupt the service on its rented bands, but can instead limit the transmit power of its users, so that it will not cause harmful interference the the services of the original spectrum owner.

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