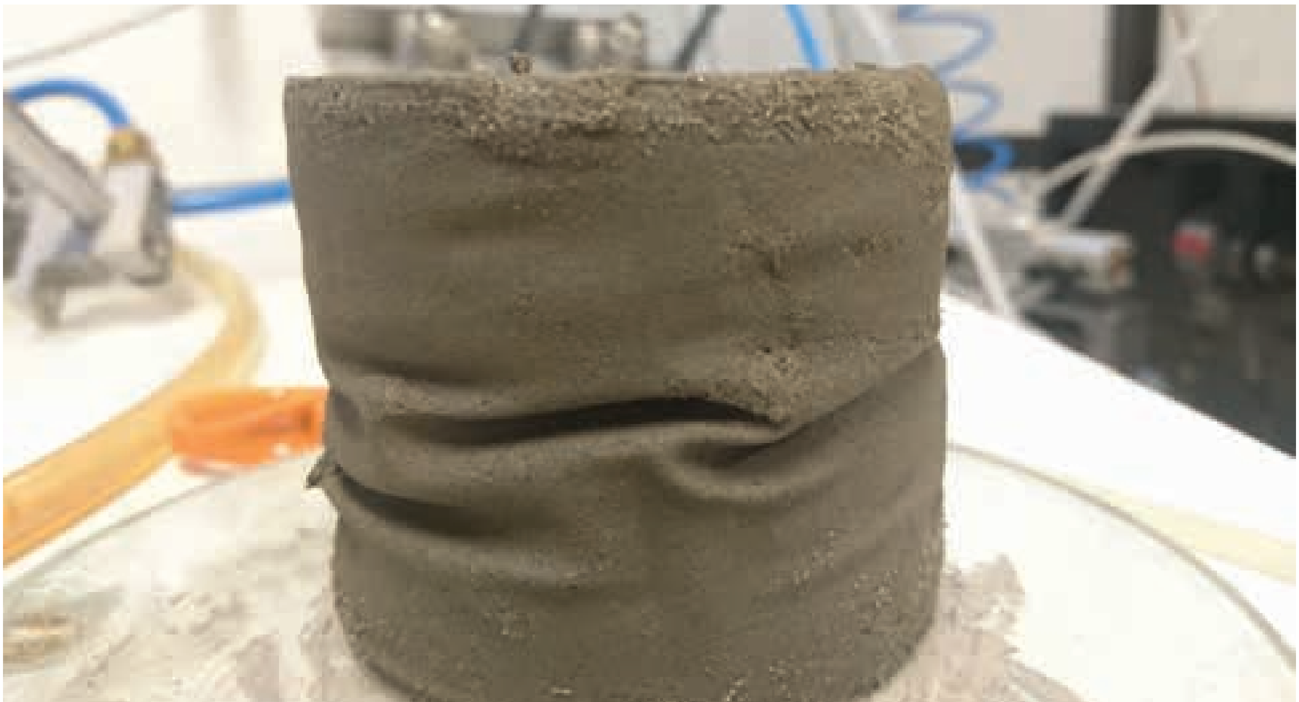
https://doi.org/10.25296/1993-5056-2019-14-1-36-42

SOIL STRENGTH CRITERION WITH ACCOUNT FOR SHEAR RESISTANCE CAUSED BY PARTICLE ENGAGEMENT



https://upload.wikimedia.org/wikipedia/commons/1/17/Post-liquefaction_testing.jpg

ANATOLY Yu. MIRNYY
Lomonosov Moscow State University, Moscow, Russia, MirnyyAY@mail.ru
Address: Bld. 1, Leninskie Gory, 119991, Moscow, Russia

Original paper
Received 14 January 2019 / Accepted 5 March 2019 / Published 29 March 2019

Abstract
Shear resistance of soil becomes vital in geotechnical design of dams and embankments, and also landslides stabilization. Historically, the Mohr-Coulomb yield criterion was used for such problems solving — it appears to be suitable for the most of fine-grained soils. But in case of gravel soils, it was noticed that they possess so-called "cohesion" although there is no physical mechanism of such behavior. This extra shear resistance of non-friction nature is caused by the particle engagement. The engagement phenomenon was usually studied by hydro engineers, but since deep excavations and heavy structures are becoming common in civil engineering, more precise calculation becomes critical. This issue is dealing with the new yield criterion for gravel soils development. The most common criteria for non-cohesive soils and the parameters they are based on are analyzed. The proposed yield criterion is based on invariant stress parameters and concerns friction, cohesion and engagement between particles. It also takes into account second principal stress by using a non-fixed sliding plane. The parameters of this criterion are physically justified and can be determined by a standard soil test. Although it still needs experimental validation, this new criterion appears to be prospective for the usage in numerical modeling, as it is universal and versatile.

Key words:
yield criterion; soil strength; shear resistance; Mohr-Coulomb criterion; Matsuoka-Nakai criterion; Duncan-Lade criterion; particle engagement

For citation:
Mirnyy A.Yu., 2019. Soil strength criterion with account for shear resistance caused by particle engagement. Engineering Geology World, Vol. XIV, No. 1, pp. 36–42, https://doi.org/10.25296/1993-5056-2019-14-1-36-42

УСЛОВИЕ ПРОЧНОСТИ ГРУНТА, УЧИТЫВАЮЩЕЕ СОПРОТИВЛЕНИЕ СДВИГУ, ВЫЗВАННОЕ ЗАЦЕПЛЕНИЕМ

МИРНЫЙ А.Ю.
Московский государственный университет им. М.В. Ломоносова, г. Москва, Россия, MirnyyAY@mail.ru
Адрес: Ленинские горы, д. 1, г. Москва, 119991, Россия

Оригинальная статья
Поступила в редакцию 14.01.2019 / Принята к публикации 05.03.2019 / Дата публикации 29.03.2019

Аннотация
Сопротивление грунта сдвигу является принципиально важным при проведении геотехнических расчетов дамб, насыпей и расчетах устойчивости откосов. Исторически для решения подобных задач использовалось условие текучести Мора-Кулона как наиболее подходящее для большинства мелкодисперсных грунтов. Однако в случае крупнообломочных грунтов было замечено, что при обработке результатов испытаний проявляется «сцепление», хотя физического механизма для такого поведения в этих грунтах нет. Это дополнительное сопротивление сдвигу, не связанное с трением, вызвано зацеплением между частицами. Явление зацепления обычно исследовалось инженерами-гидротехниками, но широкое распространение глубоких выработок и тяжелых сооружений в гражданском строительстве требует проведения более точных расчетов. В данной статье рассматривается разработка нового условия текучести для крупнообломочных грунтов. Проанализированы наиболее распространенные условия текучести для несвязных грунтов и используемые в них параметры, отмечены их преимущества и недостатки. Показано, что учет промежуточного главного напряжения оказывает принципиальное влияние на результат расчета. Предлагаемое условие текучести основано на инвариантных параметрах напряжений и учитывает трение, сцепление и зацепление между частицами. В нем учитывается промежуточное главное напряжение и используется незафиксированная площадка скольжения. Параметры данного условия физически обоснованы и могут быть определены стандартными испытаниями грунта. Несмотря на то, что данное условие требует дополнительной экспериментальной проверки, оно представляется перспективным для применения при численном моделировании, так как является универсальным и учитывает различные факторы, а так же в связи с использованием инвариантных параметров напряженного состояния и учетом промежуточного главного напряжения.

Ключевые слова:
условие текучести; прочность грунта; сопротивление сдвигу; условие Мора-Кулона; условие Мацуока-Накаи; условие Дункана-Ладе; зацепление частиц

Ссылка для цитирования:
Мирный А.Ю., 2019. Условие прочности грунта, учитывающее сопротивление сдвигу, вызванное зацеплением. Инженерная геология, Том XIV, № 1, с. 36–42, https://doi.org/10.25296/1993-5056-2019-14-1-36-42

Introduction

Under the soil strength its shear resistance is usually recognized. It is a fundamental soil parameter, and its appropriate value is vital for rational design of numerous constructions, especially soil dams. The shear resistance of soil and its strength is determined by: a) inner friction forces, depending on normal stress; b) reversible cohesion of water-colloid nature, c) rigid structural cohesion[1]. Structural cohesion is normal for rock soils and usually is not considered in disperse soils. Thus, the most rational method for fine-grained soils shear resistance description is the use of two-parametrical yield criterion.

One of the most well-known yield criteria is a Mohr-Coulomb criterion [12]:

$$\sin\varphi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 + 2c \cdot ctg\varphi}. \quad (1)$$

The Mohr-Coulomb theory considers the sliding plane that intersects the plane of the maximum and minimum principal stresses and is parallel to the axis of the intermediate principal stress. Many authors have noted that the Mohr-Coulomb theory is most suitable for solving problems of a flat stress-strain state, slopes stability problems mostly [3, 4, 6].

A significant drawback of this platform is its non-physical hypothesis — intermediate principal stress takes an infinitely large value, its effect is not considered [8]. This leads to an

[1] Maslov N.N., 1968. Soil mechanics and engineering geology basics. Vysshaya Shkola, Moscow. (in Russian)

underestimation of the strength, as this plane is the most dangerous, but is no likely to appear in complicated stress-strain state therefore its application to strength and stability calculations of the foundations of most buildings is not recommended.

Experimental studies of different soils in a wide range of stresses (from 0 to 4.0 MPa), performed by various researchers[2] [1, 7] have revealed, that the intermediate principal stress has an influence on the shear resistance, hence the yield condition needs to account for this effect.

There was an interesting yield condition (2) performed by A.I. Botkin [2, 8], who suggested that the Mohr-Coulomb condition is satisfied on the octahedral plane, equally tilted to the axes of principal stresses:

$$\tau_i^* = \sigma \cdot \mathrm{tg}\varphi_{oct} + c_{oct}, \qquad (2)$$

where $\tau_i^* = \dfrac{\sqrt{(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}}{3}$ — shearing stress on the octahedral plane; $\sigma = (\sigma_1 + \sigma_2 + \sigma_3)/3$ — normal stress on the octahedral plane; φ_{oct} — inner friction angle for this plane; c_{oct} — cohesion for this plane.

This theory takes into account the intermediate principal stress, however, also has certain inaccuracy. The magnitude of the normal stress on this plane is equal to the arithmetic mean of the three principal stresses. This decision leads to the overestimation of the normal stress. The shearing stress on this site has a small value, as calculated on the basis of deviations of the values of the principal stresses from the average, and obviously cannot be very large.

Malyshev M.V. [5] noted the «phenomenological» character of this theory, as plane selection is dictated by mathematical simplicity. An important element of this theory is the notion of the angle θ, defining the direction of shearing stress acting on the octahedral plane.

The results of the studies summarized by L.N. Rasskazov[3] [11] confirm significant influence of intermediate principal stress on the shearing resistance (up to 10°); the nonlinearity of the envelope of the Mohr circles in the majority of cases (reduction of up to 15°); the effect on the strength of the loading trajectory and Nadai-Lode parameter (change in the range of 2–4°). In addition, to date there is no possibility of accounting for engagement of particles in coarse-grained soils and the resulting increase in strength.

These phenomena are especially important in the calculation of foundations and earth structures composed of coarse-grained soils with sand or clay filler. P.I. Gordienko [5] suggested to assess the strength of coarse-grained soil by using not the φ and c parameters, but the shearing angle ψ, determined with $c = 0$. Each value of ψ has appropriate σ_1, σ_3 and σ_n values. Changing of ψ for gravel materials, depending on the stresses as per suggestion of P.I. Gordienko, can be approximately estimated by the formula (3).

$$\psi_\sigma = \psi_0 - 5\lg\frac{\sigma_n}{\sigma_0}, \qquad (3)$$

where ψ_0 — shearing angle at $\sigma_3 \to 0$, $\sigma_n = (2\sigma_1\sigma_3)/(\sigma_1 + \sigma_3)$ — normal stress on the sliding plane; $\sigma_0 = 0.2$ MPa.

Unfortunately, these relationships are markedly empirical and must be adjusted for each particular site.

At present, a large number of yield and strength conditions where developed for non-cohesive soils. Some of them, such as the Matsuoka-Nakai and Lade-Duncan criteria [9, 14], take into account the magnitude of the intermediate principal stress and are applicable at different values of Nadai-Lode parameter, but are single-parametric. Others, such as the Drucker-Prager criterion, use the same parameters as the Coulomb-Mohr, but have disadvantages, expressed in incorrect results for volume strain and dilatancy [12].

The strength criteria, introduced by Lade and Duncan [9] (4), or Matsuoka and Nakai [9] (5) were obtained by Mohr-Coulomb criteria modernization and have smooth yield surface without any angles. This yield surface deviator section form was verified by the axisymmetric triaxial compression tests. Both of these criteria are based on strength invariants, i.e. are suitable for complicated stress states. On the other side, none of them takes cohesion or engagement into account, assuming that the critical line is straight and starts from the origin of coordinates.

$$\frac{I_1^3}{I_3} = \frac{(-3-\sin\varphi)^3}{(-1-\sin\varphi)(-1+\sin\varphi)}, \qquad (4)$$

$$\frac{I_1 I_2}{I_3} = \frac{9-\sin^2\varphi}{1-\sin^2\varphi}, \qquad (5)$$

where I_1, I_2, I_3 are the corresponding stress invariants.

In this study the authors made an attempt to develop the yield criterion, taking into account all the described above regularities and phenomena. The yield condition suitable for the description of shear resistance of coarse soils should meet the following requirements:

- it should consider the influence of intermediate principal stress and type of stress state. This is achieved by consideration of the sliding plane, which is not fixed in space of principal stresses and is determined by the current values of stresses;
- take into account the physical nature of the strength of fine-grained soils;
- parameters for this condition should be invariant, i.e., should not depend on the method of their estimation.

The authors deliberately omitted the question of the shear resistance dependence of soil density, since, in practice, coarse-grained soils are often used when the density is close to maximum, and the compaction during loading is negligible.

The task of developing the yield condition can be divided into three fundamental stages. The first step is to define the most dangerous plane, where all the stresses are considered. The second step is to choose invariant parameters, corresponding to the physical meaning of the described phenomena. Finally, in the third stage it is necessary to establish the mathematical relationship between these parameters and the components of stresses at the selected plane, allowing describing the nature of the envelope of the Mohr circles observed in the experiment. This division greatly simplifies the solution to this problem, since the definition of the stress state and the identification of plane is a mathematical abstraction,

[2] Mendoza T., 1984. The strength of coarse-grained soils with sand filler in terms of spatial stress state. Ph.D. Thesis. Moscow Institute of Construction Engineers, 1984. (in Russian)

[3] Goldin A.L., Rasskazov L.N., 2001. Soil dams design. ASV, Moscow. (in Russian)

but the yield condition, on the contrary, should have reference to the nature of the phenomenon in the medium.

Choosing a sliding plane

The account of all three principal stresses is possible when considering the plane, which is determined by the magnitude of the principal stresses at the ultimate state. It is obvious that such a plane will have a different angle relative to each of axes of principal stresses. General view of such a plane is shown in fig. 1.

The position of the plane may be set by the guiding cosines of the normal vector. The module of this vector is expressed as follows (6):

$$l = \frac{\sigma_1\sigma_2\sigma_3}{\sqrt{\sigma_1^2\sigma_2^2 + \sigma_2^2\sigma_3^2 + \sigma_3^2\sigma_1^2}}. \quad (6)$$

From the consideration of triangles values of the guiding cosines can be obtained:

$$\cos\alpha = \frac{l}{\sigma_1} = \frac{\sigma_2\sigma_3}{\sqrt{\sigma_1^2\sigma_2^2 + \sigma_2^2\sigma_3^2 + \sigma_3^2\sigma_1^2}}, \quad (7)$$

$$\cos\beta = \frac{l}{\sigma_2} = \frac{\sigma_1\sigma_3}{\sqrt{\sigma_1^2\sigma_2^2 + \sigma_2^2\sigma_3^2 + \sigma_3^2\sigma_1^2}}, \quad (8)$$

$$\cos\gamma = \frac{l}{\sigma_3} = \frac{\sigma_1\sigma_2}{\sqrt{\sigma_1^2\sigma_2^2 + \sigma_2^2\sigma_3^2 + \sigma_3^2\sigma_1^2}}. \quad (9)$$

Using the known formulas of solid mechanics to determine the normal and shearing stresses on any plane we can get the expressions for these stresses:

$$\sigma_n = \sigma_1\cos^2\alpha + \sigma_2\cos^2\beta + \sigma_3\cos^2\gamma, \quad (10)$$

$$\tau_n = \sqrt{\sigma_1^2\cos^2\alpha + \sigma_2^2\cos^2\beta + \sigma_2^3\cos^2\gamma - \sigma_n^2}. \quad (11)$$

After substituting the previously obtained values of the guiding cosines (7–9) these expressions take the form:

$$\sigma_n = \frac{(\sigma_1\sigma_2\sigma_3)(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}{\sigma_1^2\sigma_2^2 + \sigma_2^2\sigma_3^2 + \sigma_3^2\sigma_1^2}, \quad (12)$$

$$\tau_n = \frac{\sigma_1\sigma_2\sigma_3}{\sigma_1^2\sigma_2^2 + \sigma_2^2\sigma_3^2 + \sigma_3^2\sigma_1^2}\sqrt{3\left(\sigma_1^2\sigma_2^2 + \sigma_2^2\sigma_3^2 + \sigma_3^2\sigma_1^2\right) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)^2}. \quad (13)$$

These values of normal and shear stresses are proposed to use in determining invariant sets of the strength parameters.

If we consider the simplest stress states for this criterion, we can see, than in case of hydrostatic compression ($\sigma_1 = \sigma_2 = \sigma_3 = \sigma$) normal and shearing stresses on the plane appear to be:

$$\sigma_n = \frac{\left(\sigma^3\right)\left(\sigma^2 + \sigma^2 + \sigma^2\right)}{\sigma^4 + \sigma^4 + \sigma^4} = \frac{\sigma^3 \cdot 3\sigma^2}{3\sigma^4} = \sigma, \quad (14)$$

$$\tau_n = \frac{\sigma^3}{3\sigma^4}\sqrt{3\left(3\sigma^4\right) - \left(3\sigma^2\right)^2} = \frac{\sqrt{9\sigma^4 - 9\sigma^4}}{3\sigma} = 0. \quad (15)$$

In the case of axisymmetric compression with Nadai parameter $\lambda = -1$ ($\sigma_1 > \sigma_2 = \sigma_3$) we have:

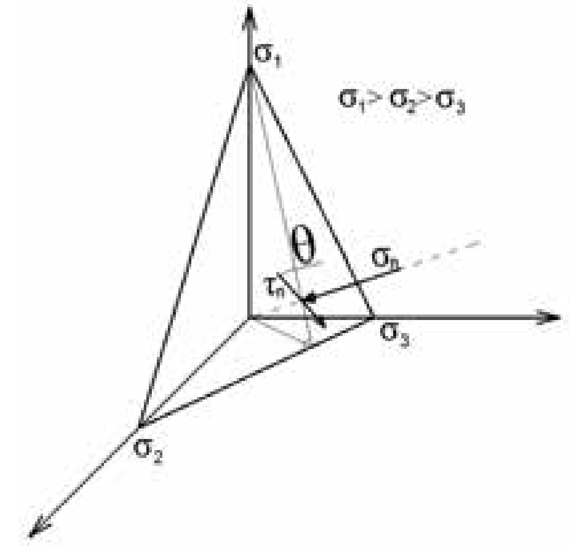


Fig. 1. General view of a suggested plane in principal stress space

Рис. 1. Общий вид предлагаемой площадки скольжения в пространстве главных напряжений

$$\sigma_n = \frac{\left(\sigma_1\sigma_3\right)\left(2\sigma_1 + \sigma_3\right)}{2\sigma_1^2 + \sigma_3^2}, \quad (16)$$

$$\tau_n = \frac{\sigma_1\sigma_3}{2\sigma_1^2 + \sigma_3^2} \cdot \sqrt{2} \cdot \sqrt{\sigma_1^2 + \sigma_3^2 - 2\sigma_1\sigma_3}. \quad (17)$$

For axisymmetric compression with Nadai parameter $\lambda = +1$ ($\sigma_1 = \sigma_2 > \sigma_3$) the result is will be similar, except for the corresponding main stress factor:

$$\sigma_n = \frac{\sigma_1\sigma_3\left(\sigma_1 + 2\sigma_3\right)}{\sigma_1^2 + 2\sigma_3^2}, \quad (18)$$

$$\tau_n = \frac{\sigma_1\sigma_3}{\sigma_1^2 + 2\sigma_3^2} \cdot \sqrt{2} \cdot \sqrt{\sigma_1^2 + \sigma_3^2 - 2\sigma_1\sigma_3}. \quad (19)$$

The expressions (17) and (19) comparison shows, that for Nadai parameter $\lambda = +1$ shearing stress on the considered plane is smaller, then for $\lambda = +1$. It correlates well with the experimental data obtained by many researchers.

Choosing the invariant strength parameters

Many years of experience in the application of the Mohr-Coulomb criterion in soil mechanics has shown that the two-parameter model is reasonable representation of the shear strength of fine-grained soils. However, studies on the shear resistance of coarse-grained soils in a wide range of stresses show poor convergence with the experimental observations. In the works of many researchers, in particular, G.G. Boldyrev, the discrepancy in the values of the strength parameters obtained by various methods is shown. This fact leads to the conclusion about insufficiency of our ideas about the nature of strength of fine-grained soils.

First and foremost this statement refers to the evaluation of internal friction through internal friction angle. The internal friction in dispersed soils is caused not only by sliding friction the Amontons-Coulomb, but the rolling friction when turning

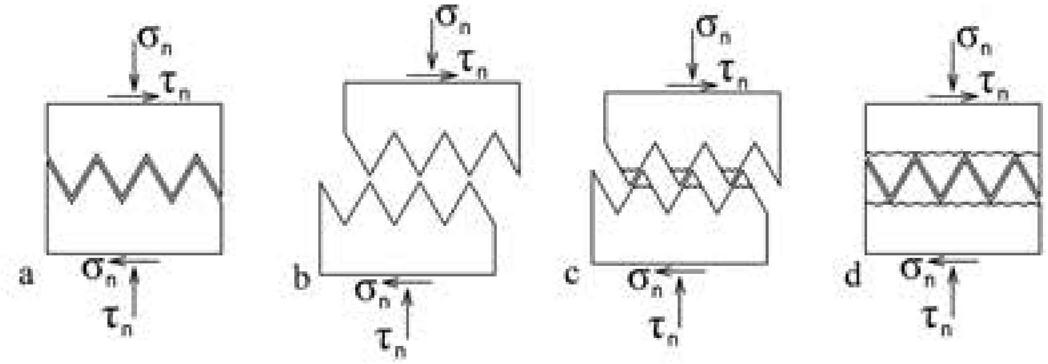


Fig. 2. Basic engagement mechanism: a — starting position; b — infinite material strength; c — finite material strength; d — no shearing resistance for material

Рис. 2. Принципиальный механизм зацепления: a — начальное положение; b — бесконечно прочный материал; c — материал ограниченной прочности; d — материал без сопротивления срезу

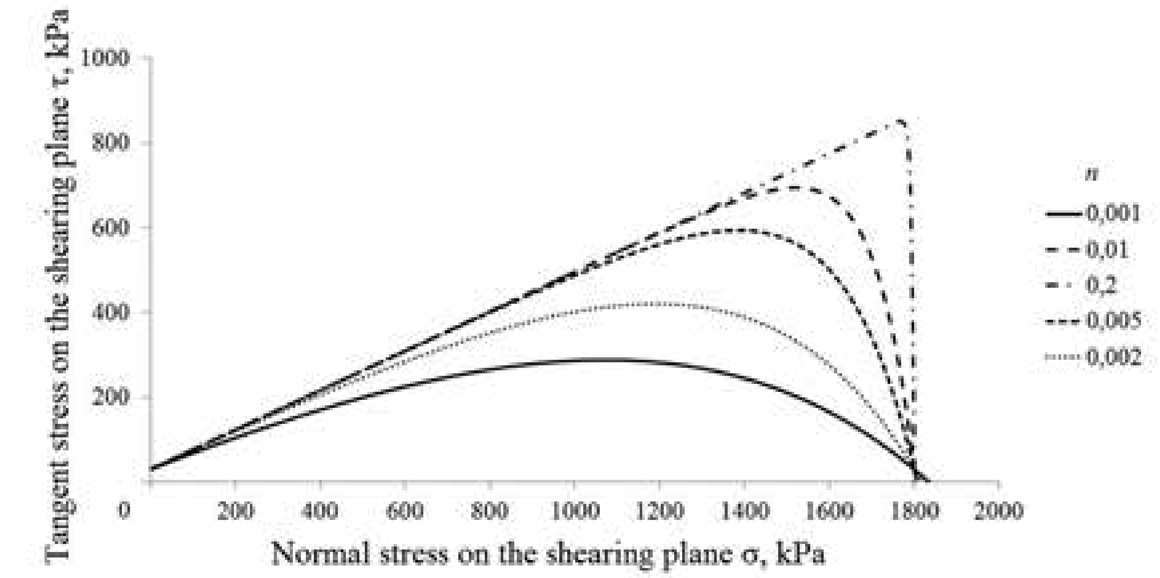


Fig. 3. Limit lines with different values of *n*

Рис. 3. Предельные линии для различных значений параметра *n*

the particles, the roughness and roundness of the particles and, consequently, their engagement. Obviously, part of these phenomena does not depend on the normal stress applied and can be related to the cohesion forces.

At the same time, the strength parameters are a function of the loading rate. For example, during a very slow loading, internal links partially recover and the sliding friction is not transformed into the rolling friction.

Thus, as the invariant of the strength parameters of the soil the following should be considered:

- the inner friction parameter, for sliding friction determined by soil mineralogy. If is considered peak strength could be accounted for. As a residual value of the natural slope angle should be taken;
- the cohesion parameter *c*, reflecting the influence of internal relations, regardless of their reversibility and applied stresses. It is constant, due to physical and chemical parameters of the soil;
- the engagement parameter, reflecting particle roughness and its influence on the shearing resistance.

The most difficult task is to identify the role of engagement in the shear strength of fine-grained soils. From physical sense of engagement, it follows that its magnitude does not depend on the compression, as even in the absence of compression with only shear forces applied there is a resistance due to engagement. The magnitude of this resistance will be constant for an infinitely strong material, as the destruction of the engagement irregularities will not occur. With decreasing strength of the material the irregularities will begin to split. In the course of the shearing there will be gradual bridging of the bumps. Finally in the case of extremely fragile material, engagement will not be observed, since the shear plane will pass through the material of the particles. This mechanism is presented in fig. 2.

At the same time, the magnitude of compression will change the amount of friction; the magnitude of the normal force to the surface of the prong is greater than the magnitude of the force normal to the cut surface due to the tilt of prongs. Chipping of prongs will occur when the shear stress exceeds the shear strength of the particles. Thus, the contribution of the engagement in shear resistance is easier to evaluate in the form of increased friction. The engagement is implemented before the start of the shearing prongs, and then begins to decline to zero with smoothing irregularities.

The invariant yield criterion development

Yield criterion is taking the form:

$$\tau^* = \sigma \cdot \mathrm{tg}\varphi \cdot \left(1 - (1+n)^{\sigma - R}\right) + c\,, \qquad (20)$$

where R — axial compression resistance of particles material, n — parameter, depending on shear resistance of particles material and obtained experimentally.

This yield criterion takes into account the shear resistance due to the engagement and its gradual decrease due to the prongs shearing.

The parameter n can be expressed, provided that we know the shear resistance of the particles, however, it is more convenient to define it on the basis of experimental data. It should be guided by the following values:

— $n = 0$ — there is no friction in shear resistance;

— $n = 0.2$ — the prongs are nearly indestructible.

On the fig. 3 there is a graph of a limit line function $\tau^* = f(\sigma)$ with different n values.

It is obvious that such view explains the definition of engagement by many authors as an additional constant component close to cohesion. This is due to the application of the linear form of the limit line. Holding such line through any three points of the curve shows initial shear strength, different from zero, which is interpreted as «engagement».

In its final form, the proposed yield criterion can be formulated in the form of three equations, allowing to determine the stress components for the sliding plane and the interaction between them:

$$\begin{cases} \sigma_n = \dfrac{(\sigma_1\sigma_2\sigma_3)(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}{\sigma_1^2\sigma_2^2 + \sigma_2^2\sigma_3^2 + \sigma_3^2\sigma_1^2} \\ \tau_n = \dfrac{\sigma_1\sigma_2\sigma_3}{\sigma_1^2\sigma_2^2 + \sigma_2^2\sigma_3^2 + \sigma_3^2\sigma_1^2}\sqrt{3\left(\sigma_1^2\sigma_2^2 + \sigma_2^2\sigma_3^2 + \sigma_3^2\sigma_1^2\right) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)^2} \\ \tau^* = \sigma \cdot \mathrm{tg}\varphi \cdot \left(1 - (1+n)^{\sigma - R}\right) + c \end{cases} \qquad (21)$$

It should be noted that the friction angle in this criterion is not numerically equal to the friction angle for other yield conditions, in particular the Mohr-Coulomb and von Mises-Schleicher-Botkin. The technique of definition of the parameters in general is similar to the conventional test methods, however, when interpreting data it is necessary to consider the range of stresses in the test. The angle of internal friction and cohesion should be determined according to the initial part of a limit line, where even in the area of stress concentration the strength of the particles is not exceeded.

For more obvious comparison of this new criterion with the others, it appears to be useful to draw the yield surface. Unfortunately, there are some mathematical difficulties in this task. Nevertheless, on the fig. 4 there are three sections of the surface with planes, normal to the lower main stress axis.

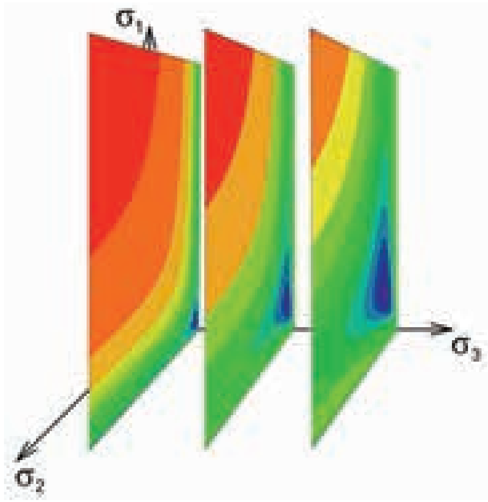


Fig. 4. General view of the new criterion yield surface sections

Рис. 4. Общий вид сечений поверхности текучести предлагаемого условия

From the drawing it can be seen, that the yield surface is pretty similar to Matsuoka-Nakai version, but the triangular form has more stretched angles. It is a disadvantage for sure: such form leads to exaggeration of the largest stress influence. In Matsuoka-Nakai criterion this problem was solved by using square roots of stresses instead[4], but this approach made cohesion consideration impossible. This problem still requires additional consideration

Conclusions

1. The most common yield criteria have limited application for coarse-grained soils, as the engagement and the decrease in shear resistance with increasing normal stress is not fully taken into account. Usually such stress level is not achieved in the civil constructions foundations, but for deep excavations and heavy structures it is vital.
2. A yield condition suitable for coarse-grained soils must consider the engagement of the particles and enable the strength of the particles as a parameter. Such approach allows describing non-linear shearing behavior of the soil.
3. The proposed yield condition can be used to describe the limit line, taking into account its nonlinearity with only to extra parameters. In addition, these parameters are of physical nature and can be determined by common test methods on the basis of experimental data.
4. Although mathematically this new criterion appears to be promising, it still needs wide experimental verification, preferably with true triaxial compression tests.

REFERENCES

1. Ataya S.M., 1965. Gravel soils shear resistance studies in triaxial machine. VODGEO, Moscow. (in Russian)
2. Boldyrev G.G., 2008. Soils mechanical properties obtaining methods. Publishing house of the Penza State University of Architecture and Construction, Penza. (in Russian)
3. Bolton M.D., 1986. The strength and dilatancy of sands. Geotechnique, Vol. 36, No. 1, pp. 65–78.
4. Duncan J.M., Wright S.G., 2005. Soil Strength and Slope Stability. John Wiley and Sons, New Jersey.

[4] Benz T., 2007. Small-strain stiffness of soils and its numerical consequences. Ph.D. Thesis. Institut fur Geotechnik der Universitat Stuttgart, Stuttgart.

5. Gordienko P.I., 1960. Some design issues of high stone and soil dams. Moscow Institute of Civil Engineering, Moscow. (in Russian)
6. Klein G. K., 1977. Structural mechanics of granular medium. Stroizdat, Moscow. (in Russian)
7. Kryzhanovskiy A.L., Mendoza T., Ukibaev E., 1985. Non-cohesive soil mixture shear resistance. Inzhenernaya geologiya, No. 2, pp. 35–41. (in Russian)
8. Malyshev M.V., 1994. Soil strength and stability of foundations of structures. Stroizdat, Moscow. (in Russian)
9. Mengcheng L., Yufeng C., Hanlong L., 2012. A nonlinear Drucker-Prager and Matsuoka-Nakai unified failure criterion for geomaterials with separated stress invariants. International Journal of Rock Mechanics and Mining Science, No. 50, pp. 1–10.
10. Mirnyy A.Yu., Merkin V.E., 2016. Choosing and estimating shear resistance parameters of gravel soils. Integration, Partnership and Innovation in Construction Science and Education, Materials of the 5th International Scientific Conference, MATEC Web of Conferences, Vol. 86, 03013, https://doi.org/10.1051/matecconf/20168603013.
11. Rasskazov L.N., 1968. Gravel soils shear resistance. VODGEO, Moscow. (in Russian)
12. Rowe P.W., 1962. The stress-dilatancy relation for static equilibrium of an assembly of particles in contact. Proceedings of the Royal Society of London. Series A, Vol. 269, pp. 500–527.
13. Ter-Martirosyan Z.G., 2009. Soil mechanics. ASV, Moscow. (in Russian)
14. Vermeer P.A., De Borst R., 1984. Non-associated plasticity for soils, concrete and rock. Heron, Vol. 29, No. 3, pp. 3–64.

СПИСОК ЛИТЕРАТУРЫ

1. Атайя С.М., 1965. Исследование сопротивления сдвигу крупнообломочных грунтов в трехосных приборах. ВОДГЕО, Москва.
2. Болдырев Г.Г., 2008. Методы определения механических свойств грунтов. ПГУАС, Пенза.
3. Bolton M.D., 1986. The strength and dilatancy of sands. Geotechnique, Vol. 36, No. 1, pp. 65–78.
4. Duncan J.M., Wright S.G., 2005. Soil Strength and Slope Stability. John Wiley and Sons, New Jersey.
5. Гордиенко П.И., 1960. Некоторые вопросы проектирования высоких каменно-земляных плотин. МИСИ, Москва.
6. Клейн Г.К., 1977. Строительная механика сыпучих тел. Стройиздат, Москва.
7. Крыжановский А.Л., Мендоса Т., Укибаев Е., 1985. Сопротивление сдвигу смеси сыпучих грунтов. Инженерная геология, № 2, с. 35–41.
8. Малышев М.В., 1994. Прочность грунтов и устойчивость оснований сооружений. Стройиздат, Москва.
9. Mengcheng L., Yufeng C., Hanlong L., 2012. A nonlinear Drucker-Prager and Matsuoka-Nakai unified failure criterion for geomaterials with separated stress invariants. International Journal of Rock Mechanics and Mining Science, No. 50, pp. 1–10.
10. Mirnyy A.Yu., Merkin V.E., 2016. Choosing and estimating shear resistance parameters of gravel soils. Integration, Partnership and Innovation in Construction Science and Education, Materials of the 5th International Scientific Conference, MATEC Web of Conferences, Vol. 86, 03013, https://doi.org/10.1051/matecconf/20168603013.
11. Рассказов Л.Н., 1968. Экспериментальные исследования сопротивляемости крупнообломочных грунтов сдвигу. ВНИИ ВОДГЕО, Москва.
12. Rowe P.W., 1962. The stress-dilatancy relation for static equilibrium of an assembly of particles in contact. Proceedings of the Royal Society of London. Series A, Vol. 269, pp. 500–527.
13. Тер-Мартиросян З.Г., 2009. Механика грунтов. АСВ, Москва.
14. Vermeer P.A., De Borst R., 1984. Non-associated plasticity for soils, concrete and rock. Heron, Vol. 29, No. 3, pp. 3–64.

INFORMATION ABOUT THE AUTHORS

ANATOLY Yu. MIRNYY

Senior research scientist of the Research Laboratory of Geological Factors Influence on Physicochemical Soil Consolidation, Faculty of Geology, Lomonosov Moscow State University, PhD (Candidate of Science in Technics), Moscow, Russia

ИНФОРМАЦИЯ ОБ АВТОРАХ

МИРНЫЙ АНАТОЛИЙ ЮРЬЕВИЧ

Старший научный сотрудник лаборатории исследования влияния геологических факторов на физико-химическое закрепление грунтов геологического факультета Московского государственного университета им. М.В. Ломоносова, к.т.н., г. Москва, Россия

Quartz sand in microscale, authors picture